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PHYSICAL REVIEW.

THE EQUILIBRIUM PRESSURE OF A VAPOR AT A CURVED SURFACE.

By Arthur A. Bacon.

Introductory and Historical.

LORD KELVIN[1] has shown from theoretical considerations that the vapor pressure for equilibrium between a liquid and its vapor is a function of the curvature of the surface of the liquid. In a closed vessel containing only a liquid and its vapor, all at the same temperature, the liquid rests with its free surface raised or depressed in capillary tubes in permanent equilibrium according to the same laws of relation between curvature and pressure as in vessels open to the air. Since, in the case of a liquid which rises in capillary tubes, the pressure is less at a capillary surface by an amount equal to the weight of a column of the vapor equal in height to the capillary height, the vapor pressure of equilibrium between the liquid and vapor must be less than at a plane surface, and similarly greater at a convex ; and if a capillary tube is placed in such a closed vessel with its free surface higher or lower than its normal capillary height, there would have to be evaporation or condensation until the surfaces were lowered or raised to capillary height. Furthermore, he says : "Without having made either the experiment or any calculation on the rate of conduction of heat in the circumstances, I feel convinced that in a very short time water would visibly rise in the tube indicated in the diagram (Fig. 1), and that provided care is

[1] Proceedings Royal Soc., Edin., Feb. 7, 1870.

taken to maintain equality of temperature all over the surface of the hermetically sealed vessel, the liquid in the capillary tube would soon take very nearly the same level as it would have were its lower end open; sinking to this level if the capillary tube were in the beginning filled too full or rising to it if (as indicated in the diagram) there was not enough of the liquid in it at first to fulfill the condition of equilibrium." He shows that the equation giving the relation between the quantities involved is,

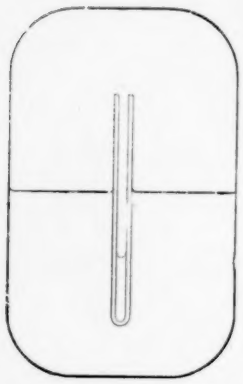

Fig. 1.

$$p = w \pm \frac{T\sigma}{\rho - \sigma}\left(\frac{1}{r} + \frac{1}{r'}\right),$$

where w is the pressure of vapor in equilibrium at a plane surface of the liquid and p the pressure of the vapor of the same liquid at the same temperature, presenting a curved surface to the vapor: T is the cohesion tension of the free surface: ρ and σ are the densities of the liquid and vapor respectively and $1/r$ and $1/r'$ the curvatures of the surfaces bounding the liquid and the vapor.

Poynting[1] suggests that this curvature of surface is accompanied by a difference of pressure in the liquid and therefore the variation of the vapor tension is presumably due to this difference of pressure.

Stefan[2] by taking Laplace's theory of capillarity discusses the condition for equilibrium at the surface of a liquid in contact with its saturated vapor and shows that the force that has to be overcome by a molecule in leaving the liquid and passing into the state of vapor is greater at a concave than at a plane surface and therefore the vapor pressure of equilibrium at a concave surface is less than at a plane surface.

Following the work of Stefan, Galitzine[3] attempts to develop the subject further. By forming various expressions to represent the energy, heat and otherwise of various changes and equating the proper quantities, he obtains a more general equation, of which Lord Kelvin's is a special case.

[1] Phil. Mag., July, 1881.
[2] Wied. Ann., 29, p. 655.
[3] Wied. Ann., 35, p 201.

Fitzgerald[1] investigates the different forces at a curved and at a plane surface, which a molecule has to overcome in order to free itself from the liquid and change to the condition of a vapor, and finds that the difference of pressure of a saturated vapor for a plane and a curved surface varies directly as the sum of the curvatures.

Schiller[2] considers the case of a cylinder with pistons at the two ends containing a liquid and its vapor. From considerations of these conditions he arrives at the conclusion that in the case of a capillary tube with radii of curvature r_1 and r_2 with a capillary constant T, the difference of pressure is

$$\pm T\left(\frac{1}{r_1}+\frac{1}{r_2}\right)\frac{\sigma}{s-\sigma}$$

where σ and s are the vapor and liquid densities respectively. This is seen to be identical with Lord Kelvin's equation.

Warburg,[3] on the basis of Carnot's principle, equates the expressions of work done in increasing the size of a drop of water at the expense of another drop, first against the capillary force, at constant temperature and without evaporation or condensation; and second, by considering evaporation and condensation alone. The equation of these expressions produces an equation identical with Lord Kelvin's. Thus from theoretical considerations it is well shown that from whatever standpoint the question is approached, the pressure of equilibrium of a vapor in contact with its own liquid should be a function of the curvature of the surface. The object of this investigation is the experimental observation of the phenomenon and a rough determination of the rate at which a condition of equilibrium would be established when liquid in capillary tubes is raised or depressed from capillary height.

Preliminary Experiment.

The form of experiment was similar to that suggested by Lord Kelvin, of a capillary tube with the liquid raised or depressed from the height at which it would stand naturally if there were hydrostatic communication between all parts of the system. The appa-

[1] Phil. Mag. (5), 8, p. 382, 1879.
[2] Wied. Ann., 53, p. 396.
[3] Wied. Ann., 28, p. 394.

ratus consisted of four capillary tubes, three with closed bottoms filled respectively to about capillary height, 5 cm. above and 5 cm. below the same and the fourth open to indicate capillary height. These were placed in a jar about five centimeters in diameter and thirty centimeters high, fitted with a ground-in glass stopper so arranged that the stopper could be covered with mercury to form a seal. The stopper terminated in a glass tube connected with a mercury gauge to measure the pressure and beyond this was hermetically sealed after the air had been pumped from the jar. To preserve constant temperature over the whole apparatus, the jar was placed in a tank of water about 30 cm. square and 45 cm. high. The water was kept constantly stirred by a ring of sheet copper, 10 cm. internal diameter and 20 cm. external, surrounding the jar and raised and lowered at the rate of eight or ten times per minute. In two opposite sides of the tank glass windows were set allowing readings to be taken by transmitted light. The apparatus was set up in the constant temperature room of the Wilder Physical Laboratory. The liquid used was ether. This was to obviate the almost insurmountable difficulty of obtaining perfectly pure surfaces of water and glass. Ether, furthermore, has a vapor density about four times that of water, so that the difference of pressure between two levels, due to the weight of a column of vapor of the same height as the difference of level, would be increased. The lower surface tension of ether is a disadvantage, but is more than compensated by the advantage gained in vapor density. The ether was distilled over sodium and had a constant boiling point of 35.4° C.

The experiment was continued in this way for nearly eight months, readings being taken every few days on a kathetometer about a meter from the tank. Condensation was seen to take place in all the tubes. During seven months the increase in height of the surface which was below the level of the liquid in the jar was a little less than a millimeter; of the surface above the level of the liquid in the jar, between 9 and 10 mm.; and of the surface near capillary height between 10 and 11 mm. The small amount of condensation at the lowest level may have been due to a greater distance from the end of the tube to the surface of the liquid in the tube. At the end of this time heavy oil was added to the surface of the water in

the tank to prevent evaporation which might be producing a cooling effect at the top. This was followed by a decrease in the rate of condensation, but there still was condensation in every case as before. Ether began to condense also in the pressure gauge outside the tank of water. The results in almost every respect were at variance with the theory and the only conclusions to be drawn from the experiment were the necessity of more constant temperature conditions and the effect of the oil on the surface of the water in the tank.

Apparatus.

The form of apparatus finally adopted was as follows: Within the jar, as shown in Fig. 2, were four tubes, internal diameter 1.58 mm., all closed at the bottom except the one to indicate capillary height. No. 1 was filled so that the free surface was 4.51 cm. above capillary height and 2.49 cm. from the end of the tube. No. 2 had its free surface 4.19 cm. below capillary height and 4.29 cm. from the end of the tube. No. 3 had its free surface 0.49 cm. above capillary height and 3.80 cm. from the end of the tube. No. 4 indicated capillary height. No pressure gauge was used and the jar after being pumped out was hermetically sealed near the stopper. A mercury seal was used around the stopper as before. Ether was again used as the liquid. A larger tank of water was used, being about 45 cm. square and 60 cm. high. The jar was raised on a platform, so as to occupy a central position in the tank. The same form of stirrer was used as before. Windows were inserted in the sides of the tank as before, for taking the observations. The tank was then surrounded, as shown in Fig. 3, by a jacket of fine shavings to a thickness of 35 cm. on all sides and on top and bottom, a wooden cover being placed over the tank of water and the two rods which moved the stirrer passed through brass tubes. For the openings through which to take readings, boxes were constructed with a cross section slightly larger than the windows and with five compartments with plate glass partitions. The readings were taken as before on a kathetometer, except that they were taken from outside the room and only three or four times during the progress of the experiment was the room entered and then only to attend to the water motor driving the stirrer or to adjust the lights for better

observations. Three sixteen candle power incandescent lamps were placed in front of the opening opposite the kathetometer to furnish the light for taking the readings. A thermometer was placed in

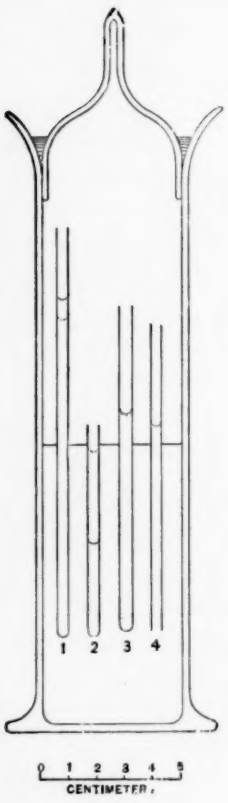

Fig. 2.

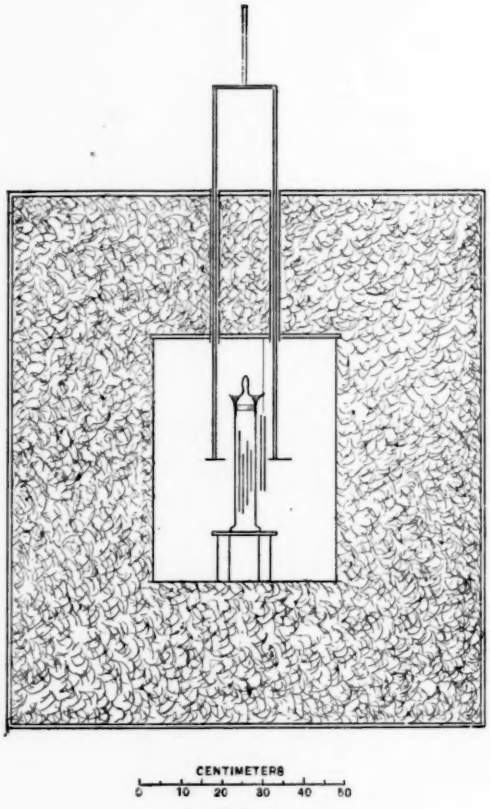

Fig. 3.

the water bath where it could be read by the telescope of the kathetometer.

OBSERVATIONS.

The observations taken were the differences in level of the meniscuses and the temperature. These are given in the table and plotted in Fig. 4. At the end of nine months the stirrer was stopped and the apparatus remained undisturbed for two months more; the last readings being a single observation just before the apparatus was taken down. The reading in tube No. 2 of the last observation is not exact to within half a millimeter as the meniscus

was obscured by the frame holding the tubes. During the time the stirrer was acting the rate of evaporation from the highest surface was only about one tenth that of the condensation in the lowest; and during the same time the level of the meniscus which was less than a centimeter above capillary height remained practically constant. However, after the stirrer was stopped the rate of evaporation from the upper tube was much increased and the level nearest capillary height was lowered, as would be expected. The question naturally arises as to which arrangement gave the better temperature conditions, *i. e.*, whether the action of the stirrer intro-

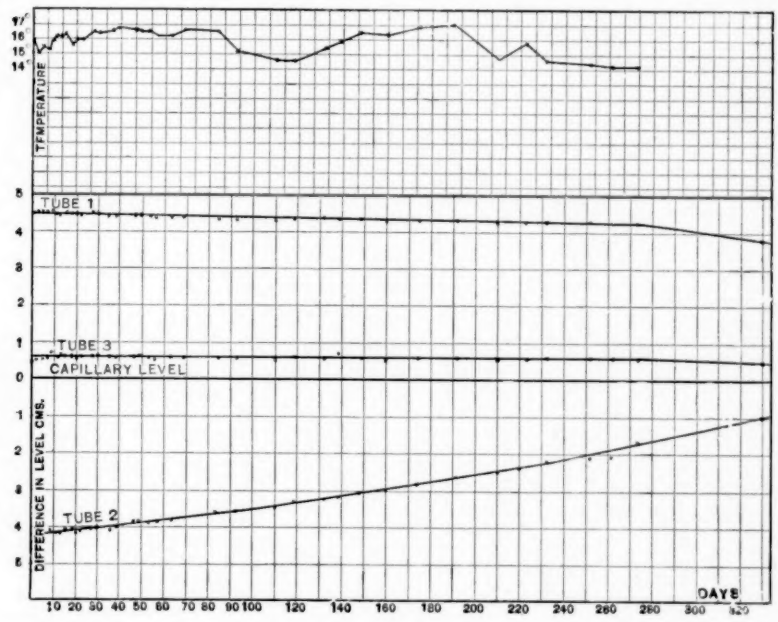

Fig. 4.

duced a temperature gradient, the water being cooler at the top, or when standing undisturbed the top of the water became warmer. The author is inclined to the former. When the apparatus was set up oil was added to the surface of the water in the tank, but when the apparatus was taken down it was found that the oil had been drawn up by the rods connecting with the stirrer and had become dried in the tubes through which these passed as well as somewhat spread around on the shavings at the top, so that the water surface

had become exposed and water was drawn up on these rods and when exposed to the room it had evaporated, and the rods descended into the water cooler, thus cooling the water at the top. It would seem, therefore, that more even temperature conditions could be maintained without the use of the stirrer, unless this difficulty could be obviated. The fact that after the stirrer was stopped the rate of evaporation from the highest surface was less than the condensation

Days.	Tube 1.	Tube 2.	Tube 3.	Temperature.
0	4.51	4.19	0.49	15.9
2	4.53	4.18	0.53	15.0
5	4.50	4.18	0.56	15.4
7	4.50	4.18	0.59	15.2
9	4.52	4.06	0.67	15.9
11	4.46	4.14	0.59	16.2
13	4.45	4.13	0.63	16.1
15	4.48	4.05	0.61	16.4
18	4.46	4.06	0.61	15.6
20	4.44	4.15	0.58	15.9
22	4.42	4.09	0.62	15.9
28	4.47	4.03	0.61	16.5
30	4.45	4.01	0.63	16.4
35	4.42	4.09	0.60	16.5
39	4.41	4.00	0.59	16.8
47	4.42	3.88	0.62	16.6
49	4.43	3.85	0.62	16.5
53	4.40	3.87	0.59	16.5
56	4.38	3.85	0.56	16.2
63	4.39	3.81	0.60	16.2
69	4.40		0.59	16.6
84	4.36	3.61	0.59	16.5
92	4.34	3.57	0.61	15.2
110	4.34	3.43	0.57	14.6
119	4.36	3.32	0.60	14.6
131	4.36	3.23	0.58	15.3
139	4.35	3.13	0.70	15.8
148	4.34	3.06	0.60	16.5
160	4.32	2.97	0.57	16.3
174	4.32	2.81	0.60	16.8
191	4.33	2.65	0.60	17.0
210	4.28	2.51	0.58	14.6
223	4.29	2.36	0.58	15.8
232	4.30	2.26	0.60	14.6
251	4.29	2.10	0.60	14.4
261	4.27	2.03	0.60	14.2
273	4.27	1.86	0.59	14.2
330	3.80	1.00	0.50	

in the lowest may have been due to the cooling effect of evaporation at the surface of the water in the tank, there being no oil on the surface then and the wooden cover not fitting tightly.

The points to be noticed in regard to the results are:

1. There is substantial agreement with the theory.

2. The process of adjustment to the condition of equilibrium is extremely slow. Even taking the case where the rate was most rapid, in tube No. 2, it would take more than a year to change the level as much as four centimeters. In view of this fact it does not seem probable that in cloud formation the growth of larger drops at the expense of smaller ones can be caused as much by evaporation and condensation due to the different equilibrium pressure demanded by the different curvatures, as it is by the uniting of smaller drops with each other to form larger ones. Especially would this seem so as the presence of the air in the ordinary atmosphere would tend to make the rate of evaporation and condensation slower than in the case covered by the experiment.

In conclusion the author desires to express his indebtedness to Professor Ernest F. Nichols at whose suggestion the problem was undertaken and under whose guidance and assistance it was carried on. Also to Professor Gordon F. Hull for further advice and suggestion. Owing to unavoidable absence nearly all the readings during the final form of the experiment were taken by Mr. J. A. Brown, assistant in the laboratory, to whom acknowledgments are due. Thanks are also expressed for the use of the constant temperature room of the Wilder Physical Laboratory of Dartmouth College for a period of over two years.

HOBART COLLEGE,
GENEVA, N. Y.

ON THE COEFFICIENT OF EXPANSION OF QUARTZ.

BY HARRISON MCALLISTER RANDALL.

THE important part played by quartz in many scientific investigations has made a determination of its physical constants a matter of considerable importance. Among the determinations of the coefficient of thermal expansion, those by Fizeau [1] in 1864, Benoit [2] in 1888, Reimerdes [3] in 1896 and Scheel [4] in 1902 may be cited as the most important. The optical method devised and used by Fizeau has formed the basis of the methods employed in all the subsequent investigations. The range of temperature employed by Fizeau extended from 2° to 60°, that of Benoit from 6° to 80°, that of Reimerdes from 5° to 220°, and that of Scheel from 16° to 100° C. In addition to these investigations, there have been made rough determinations, intended simply to show the characteristics of the expansion of quartz by Le Chatelier [5] 1889, using a photographic method, and by Sahmen and Tammann [6] in 1903 by means of a self-registering dilatograph. Le Chatelier's observations extended from 15° to 1000°, and those of Sahmen and Tammann to 700°.

In the present investigation the method of Fizeau has been employed for a range of temperature of approximately 500° C. The series of determinations first to be described was made in 1901, and, for reasons to be stated subsequently, it seemed desirable to repeat the work with such changes in the apparatus as would insure the elimination of certain errors possible with the first arrangement. Opportunity to do this was not found till the present year, when a redetermination under quite a different set of conditions was made.

[1] Ann. de Chem. et de Phys., 2, 1864, p. 143; 8, 1866, p. 335; Comp. Rend., 58, p. 923; 62, p. 1133.

[2] Trav. et Mém. du Bureau Int. Poids et Mes., I., 1881, VI., 1888.

[3] Inaug. Diss., Jena, 1896.

[4] Ann. der. Physik, 9, 1902, pp. 837–853.

[5] Comp. Rend., 108, p. 1046.

[6] Ann. der Physik, 10, 1903, p. 879–889.

Method and Apparatus.

The optical method for measuring small expansions by the displacement of interference fringes, used in this investigation, is a modification of the method used by Fizeau, and later employed by Benoit. The apparatus (Fig. 1) for the measurement of the expansion of quartz is that first suggested by Pulfrich,[1] and used by Rei-

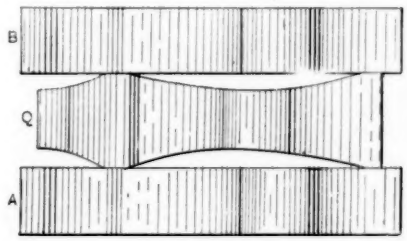

Fig. 1.

merdes in his work on the coefficient of expansion of quartz. *A*, the bed-plate, is a quartz plate 38 mm. in diameter and 7 mm. thick. Its upper surface, cut perpendicular to the principal axis of the quartz, is plane polished. The lower surface, cut parallel to the upper, is left unpolished. *Q* is a quartz ring approximately 1 cm. long, 34 mm. in external diameter and 25 mm. in internal diameter. It has been made with its axis accurately parallel to the principal axis of the quartz. Each of its ends has been ground away so as to leave three equidistant feet as shown in the figure. This ring is placed upon plate *A* and upon the ring plate *B* is placed. Plate *B* is of the same material and size as plate *A* and its surfaces, plane polished, and perpendicular to the principal axis of the quartz are inclined at an angle of 20′. There is a fourth quartz plate, *C*, about 2 mm. thick, and with a diameter of 2.2 cm. cut with its upper surface perpendicular to the principal axis of the quartz. This surface is plane polished while the under surface, unpolished, has been ground away so as to form three feet equidistant from the original surface. Plate *C* is placed inside the quartz ring upon plate *A*. One of the feet of plate *C* was ground away until the upper surface of the plate and the lower surface of *B* were properly inclined to

[1] Zeitschr. für Krystall., 31, p. 372, 1899.

give interference fringes of the desired width. These fringes were found to be perfectly straight and equidistant, showing that the reflecting surfaces were truly plane. The quartz ring Q and the small quartz plate C were furnished by Carl Zeiss, Jena, and the two larger plates A and B, by Breashear. For convenience of observation, the quartz plate B bears at the center of its lower surface a small circle which is sharply focussed by the telescope and furnishes a fixed point with reference to the fringe system. Each dark band of the fringe system represents a definite thickness of the air layer between the two reflecting surfaces. The thickness of this air layer under any dark band differs from its thickness under an adjacent dark band by $\frac{1}{2}\lambda$, where λ is the wave-length of the light used. When the system has been heated, the two reflecting surfaces are separated owing to the expansion of Q and each band, representing as it does a definite thickness of the air layer, has to move toward the thin edge of the air layer. The number of such bands passing the small circle on the cover plate represents the change in thickness of the air layer directly under the circle and thus measures the expansion of the quartz ring Q. The total expansion of the quartz for any temperature interval $(t_2 - t_1)$ may be represented by $f \times \lambda/2$ where f is the number of dark bands passing the small reference circle of the cover plate, and λ the wave-length of the light used. The coefficient of expansion, α, is therefore given by the equation

$$\alpha = \frac{f \cdot \frac{\lambda}{2}}{L(t_2 - t_1)}$$

where L is the effective length of the quartz, that is, the length of the ring minus the length of the thin plate inside the ring.

APPARATUS.

The apparatus was set up in one of the basement rooms of the Physical Laboratory, a brick pier, surmounted by a sandstone slab, served as a support for part of the apparatus, while the remainder was supported directly by the asphalt floor. Fig. 2 shows a cross-section of the apparatus arranged for work. D represents an Abbé-Fizeau interferometer, P, a porcelain tube with a brass chamber

attached to its lower end in which the expansion apparatus shown in Fig. 1, is placed; O, an oven heated electrically and U, an adjustable support for the oven. The illuminating apparatus is not shown in the figure.

The Interferometer.

The first Abbé dilatometer was made in 1884 and is described by Weidmann.[1] The instrument used in these measurements was made by Carl Zeiss, Jena, and has the many improvements introduced by Pulfrich,[2] and owing to its wide range of applications it may be more justly termed an interferometer. It consists of an autocollimating telescope mounted upon a strong pedestal. Light from a point source is focussed upon a small totally-reflecting prism, p, (Fig. 2), which is set into the telescope tube through a small opening, and occupies practically half the field. There are adjustments by which this prism may be turned about a vertical axis or about a horizontal axis perpendicular to the length of the telescope. From this prism as a source, the light is reflected to the lens L_1, which is mounted in a tube sliding within the telescope tube, and is so adjusted that the prism p is at its principal focus. The light emerging from L_1 is then a parallel beam reflected downward by the totally reflecting prism T. Prism T is mounted in a tube sliding upon the outside of the telescope tube and can consequently be turned to reflect the light in any direction perpendicular to the axis of the telescope. It can also be pushed in or out in a line with the telescope tube through a considerable range.

The light, after traversing the tube P, falls perpendicularly upon the interference apparatus and interference fringes are produced. If the reflecting surfaces of the interference apparatus were situated in the focal plane of the lens L_1, the rays proceeding from the interference fringes, considered as objects, would, after passing through the lens L_1, be parallel. Passing the free half of the telecope tube at the prism p, they would be brought to a focus at N in the focal plane of the lens L_2. The size of the heating apparatus makes it impossible to have the reflecting surfaces of the quartz apparatus in the focal plane of the lens L_1. They have to be at a

[1] Wied. Ann., 38, p. 453, 1889.

[2] Zeitschrift für Instrumentenkunde, 1893, pp. 365, 401, 435; 1898, p. 261.

little greater distance. The lens L_2 has sufficient adjustment, however, to bring the rays to a focus at N where is placed an adjustable horizontal slit. The image formed at N is viewed through the lens L_3. If the fringes do not appear vertical, they may be made so by rotating the Dove's prism E, using the spindles provided for that purpose. When light of two or more wave-lengths is used, the

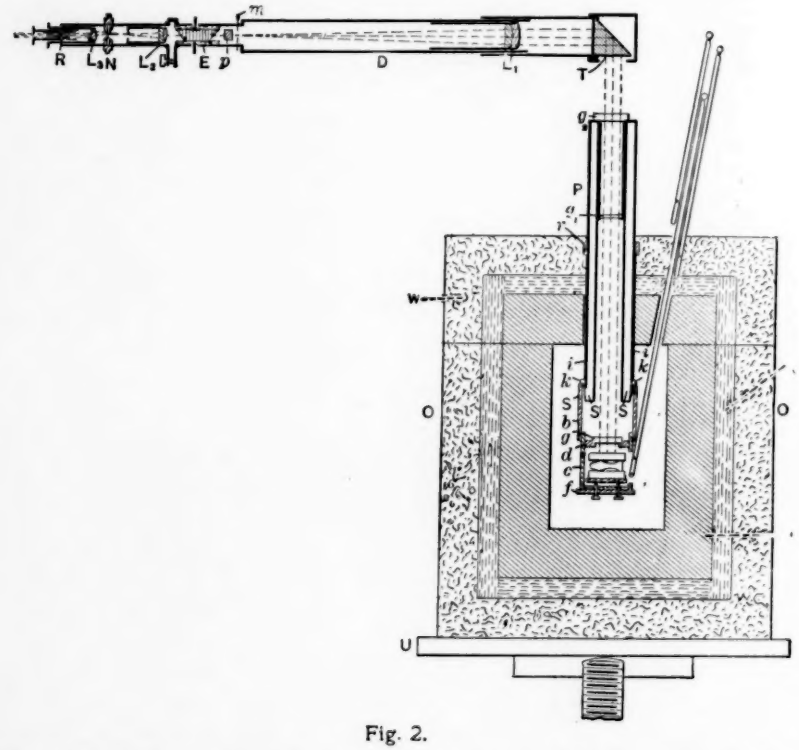

Fig. 2.

fringes due to each wave-length can be observed by inserting in the eye-piece the direct-vision spectroscope R so that the spectral colors lie one above another. On closing the adjustable slit, N, the spectral images of this slit now appear as horizontal bands whose width depends on the width of the slit, and are crossed by vertical interference bands. An iris diaphragm, m, placed close by the prism p on the side towards the lens L_1 is operated by a spindle projecting through a slit in the tube.

Immediately behind the slit N are situated two vertical cross-

wires, the distance between them being a little less than the apparent diameter of the small circle in the center of the quartz cover plate. The observing telescope is so mounted on the collimating tube as to have two independent angular movements, one vertical, the other horizontal. By the vertical movement it may be so tilted as to receive light from that part of the cover plate B, immediately about the reference circle. In this case the image of the reference circle appears in each of the colored bands. With the horizontal movement the cross-wires can be made to move apparently across the field parallel to themselves. The amount of this latter movement is measured by means of a micrometer screw, the head of which is divided into 100 equal divisions.

The Illuminating Apparatus.

Light from an H form Geissler tube is focussed by a lens upon the prism p. The tube and the lens, so mounted as to admit of easy adjustment, are carried by an arm extending out at right angles to the collimating tube. The Geissler tube contains a drop of mercury and hydrogen gas under a pressure of a few millimeters. A slight warming of the mercury globule causes the formation of a sufficient amount of mercury vapor to produce brilliant mercury lines.

The Supporting Tube P (Fig. 2).

The tube which supports the quartz apparatus in the oven is of very hard white porcelain. It is 27 cm. long, has an internal diameter of 2.7 cm. and its walls are 9 mm. thick. Tube P is supported in a vertical position directly beneath the totally reflecting prism T of the interferometer. A thick brass ring is closely fitted around the top and into this ring and at right angles to the tube is screwed an iron rod which in turn is clamped to the stone pier. This method of support proved entirely satisfactory, being sufficiently rigid and yet permitting easy change of position of the tube. At the lower end of the tube P is a shoulder, s, which serves to support the brass chamber in which the quartz apparatus is placed.

A brass ring is fitted around the tube P directly above the shoulder s, its thickness permitting it to extend slightly beyond the shoulder. The brass tube b is fastened by screws to this ring.

Tube *b* is 6 cm. in diameter and 5.5 cm. long. Into it is screwed a second tube *c*, 5.4 cm. long and 5.4 cm. in diameter. A thick brass ring *d* of internal diameter 2.5 cm., is fitted closely into the top of tube *c*. A shoulder is turned in the upper surface of ring *d*, around the inside of the proper diameter to hold tightly a circular glass plate, *g*. A bottom, *f*, is screwed into the tube *c*, through which project three equidistant leveling screws. A thin brass plate rests on the top of these leveling screws. To keep the brass chamber rigidly firm during all the temperature changes the following device was adopted. At a distance of 9 cm. above the shoulder *s* a brass ring *r* is fitted around the tube *P*. This ring is held fast by screws extending slightly into holes drilled in tube *P*. Around the top of the brass ring which rests on the shoulder *s* are placed several rings of asbestos. A thin walled tube *i* is cut just the right length to press firmly on the ring *k*, when put in place between the rings *r* and *k*. This pressure, which holds the supported chamber in position, is maintained upon increase of temperature by the expansion of the tube *i*. The asbestos rings, by their slight yielding, prevent rupture of the porcelain tube. Between the brass disks *d* and *f*, the tube *c* is half cut away so as to form a large window for the introduction of the quartz apparatus. In addition to the glass plate *g*, two other plates g_1 and g_2 are inserted, as shown in the figure, for the exclusion of cold air from above and the prevention of convection currents from below.

THE OVEN.

The central portion of the electrical heating apparatus consists of a cylindrical cup of cast iron, *I*, 23.8 cm. high, 22 cm. in external diameter, 5 cm. thick on sides and bottom, and weighing, with its two semicircular iron cover plates, about 140 pounds. Upon a sheet of asbestos paper placed around the cylinder is wound, non-inductively, No. 28 iron wire, the turns being about 3 mm. apart. A second sheet of asbestos is wound around the cylinder on top of the first layer of wire, and upon this is wound a second layer of wire of the same number of turns as the first, and in the same way a third layer is added above the second. Asbestos, *A*, is then wound around the cylinder to a thickness of two centimeters. On

the elevating stand, *U*, is placed a layer of asbestos 34 cm. in diameter and 1.5 cm. thick; upon this are placed three equidistant porcelain insulators about 4 cm. high, which carry a second asbestos layer 3 cm. thick and 22 cm. in diameter, upon which is set the iron cylinder. The space about the insulators is loosely packed with mineral wool, *W*. A sheet-iron cylinder 34 cm. in diameter, and equal in height to the combined height of the insulators and cast-iron cylinder, is next placed symmetrically around the iron cylinder, and the 4 cm. air space between is loosely packed with the mineral wool.

The sheet-iron cylinder is protected on the outside by a few layers of asbestos. The semicircular cover-plates of the iron cylinder are 5 cm. thick, pierced at the center to admit the porcelain tube freely, and are planed smooth on their under sides, as is also the top of the cylinder, to ensure perfect contact. One of the two plates has a radial slot for the passage of the thermometers. The sheet-iron cylinder is surmounted by a divided cap similar in shape to the semicircular cover-plates, and provided with asbestos and mineral-wool insulation corresponding to that in the other parts of the apparatus.

The conditions of the experiment require that the temperature of the quartz apparatus be maintained constant for long intervals of time. The oven seems well adapted to this end, since the large mass of metal employed undoubtedly renders the distribution of heat more uniform and tends to minimize sudden slight changes of temperature due to small variations in the heating current. A thermometer placed in a hole drilled in one of the cover-plates showed that while the temperature of the cover lagged behind the temperature of the air in the interior of the cylinder during the heating, it very quickly assumed practically the same temperature as soon as the latter was maintained constant. The excellent thermal connection between the covers and the cylinder is probably of value in compensating for loss of heat through the porcelain tube, since the covers encircle the tube closely at its middle point, and this part is quite likely to be the point of highest temperature in the tube.

The excellence of the heat insulation is shown by the fact that the outside of the sheet-iron cylinder was not uncomfortably warm for the hand, when the temperature inside was 500° C. An asbestos

screen, mounted half way between the top of the oven and the telescope of the interferometer, afforded ample protection against disturbing effects due to heating the interferometer.

The current used for heating was the 220-volt direct current furnished by the University lighting and power plant. For low temperatures and small currents a milliammeter was in circuit, to measure and aid in regulating the current with the adjustable resistances. For higher temperatures an ammeter reading directly to tenths was used.

The support which carries the oven has a vertical adjustment with a screw movement sufficient to lift the oven from a position several centimeters below the interference chamber, to one which brings the interference chamber to the center of the iron cylinder. The support is rigid enough to prevent all lateral movement and carries the binding posts to which the ends of the heating coils are attached.

THERMOMETERS.

Four thermometers were used for the complete range of temperature. From room temperature to 200° C. two thermometers made by Haak, Jena, were used. They read directly to $\frac{1}{5}$° and were calibrated at the Reichsanstalt in 1898. The scales of these thermometers extend from — 20° C. to 105° C., and from 90° C. to 210° C., respectively. A third thermometer, having a scale from 0° C. to 360° C., graduated in degrees, was used for the range between 200° C. and 350° C. This thermometer was calibrated at the Reichsanstalt in 1899. The fourth thermometer, graduated in degrees, is provided with a short scale at 0° C., and, after an enlargement of the bore, extends from 195° C. to 550° C. This thermometer was calibrated at the Reichsanstalt in 1897. During the progress of the investigation, the fixed points of these thermometers were determined a number of times and the necessary corrections applied.

THE SPHEROMETER.

The spherometer used to measure the lengths of the quartz ring and the small quartz plate, was made by the Geneva Society and reads directly to 0.001 mm.

Method of Setting Up Apparatus.

The Geissler tube is adjusted in its holder until a bright image about 2 mm. in diameter is formed by the focussing lens upon the face of the prism p. The adjustment is properly made when the lens L_1, viewed through the prism T turned sidewise, is seen to be uniformly illuminated. Finally, after the apparatus is all set up, quite often it is possible to improve the appearance of the fringes by a slight change in the position of the tube, using the slow motion attachment provided for this purpose. The porcelain tube P is clamped in a vertical position directly below the prism T, and the bed-plate A placed in position on the thin brass plate supported by the leveling screws. To cause the beam of light incident upon A to retrace its path after reflection, the various adjustments possible are tried in turn. These adjustments consist in leveling the plate A by means of its supporting screws, the sliding of the interferometer as a whole on the pier, the tilting of the instrument by means of its leveling screws, and the combined turning and sliding in or out of the tube containing the totally reflecting prism T. When properly adjusted the circular field will be uniformly illuminated.

The quartz pieces having been carefully cleaned, the small quartz-plate is placed in the center of the bed-plate and the quartz ring symmetrically around it. The quartz cover-plate, with the reference circle down, is placed on top of the ring. While the quartz pieces are being thus arranged, care is taken to note their relative positions with respect to one another, and to see on which side the thin edge of the air layer lies. When viewed through the instrument, interference fringes are usually visible; if not, a slight adjustment of the interferometer will invariably produce them. When the adjustments are made and the fringes appear erect and at their brightest, readings are taken upon two adjacent dark bands to determine the width of the fringes. To avoid all possibility of particles of dust or lint being between any of the surfaces supposed to be in contact, the quartz apparatus is repeatedly taken down, the parts cleaned and remounted in the same relative positions, the width of the bands being measured each time. When several successive settings give the same band width this source of error is supposed to have been avoided.

It is then necessary to get the reference circle in the center of the field, so that, should it become necessary to erect the fringes, this circle will rotate about its own center. This is most readily done by sliding the interferometer slightly about on the pier, though sometimes it becomes necessary to move the quartz apparatus in the brass chamber.

The direct-vision spectroscope is then placed in the observing telescope and adjusted till the vertical cross-wires are most distinct. The slit is narrowed till the spectral colors, when separated, appear as horizontal bands of about the width of the reference circle and crossed by equidistant dark lines. If the reference circle does not lie within these bands, it can be brought there by the vertical adjustment of the observing telescope.

To avoid any accidental fall of the quartz apparatus through the window into the iron cylinder, there is fastened over the window a piece of wire gauze in which slits 1 cm. wide and equal in length to the height of the window are cut. There is left between these slits just enough of the gauze to hold the piece together, so that this protection in no way interferes with the free circulation of the air between the brass chamber and the cylinder.

The thermometer is next adjusted with its bulb on a level with the quartz apparatus and opposite an opening in the gauze. At first the thermometer hung vertically. In this position the bulb was about 2.5 cm. from the quartz expansion apparatus. Later the thermometer was tilted so that its bulb entered the edge of the window through one of the slits in the gauze. In this position the bulb was within 1.5 cm. of the quartz. No change in the results was apparent from the change in position of the bulb of the thermometer. The thermometer is supported by suitable clamps fastened to the stone slab of the pier.

The heating apparatus is raised until the chamber containing the quartz is symmetrically situated within it. The covers previously described are placed in position and the length of the thermometer stem in the insulation noted. A second thermometer is mounted next the first with its bulb half through the insulation to give the average temperature of the mercury thread in the insulation. A glass tube, closed at its upper end, about 3 cm. in diameter, to

shut off air currents, is put about the thermometers, each of which is provided with a small thermometer to get the average temperature of the mercury columns above the insulation. The mounting of the apparatus is then complete. Generally it can be accomplished without any change in the position of the fringes and reference circle with respect to the vertical cross-wires, thus showing that the quartz apparatus has not been disturbed.

The apparatus is then left to stand over night and acquire a stationary temperature. The next morning at intervals of a quarter of an hour, the temperature of the quartz apparatus is determined and micrometer readings on the position of some one of the dark bands are taken. If for one hour these do not change materially, a set of micrometer readings is taken on the fringe system in the manner shortly to be described. The thermometers are read as is also the barometer. A current of about 7 amperes is then sent through the heating coils, an ammeter and adjustable resistance being in the circuit. With this current the temperature of the oven increases 20° C. in about half an hour. The current is then turned off till the thermometer shows that the temperature inside has become stationary, then a current which will maintain this temperature is found by trial. This current is kept constant for three or more hours. During the last two hours the fringes generally appear quite stationary and micrometer readings are taken every quarter of an hour on some dark band. When the position of the band has not changed for an hour the condition of equilibrium is considered to have been attained and a second set of readings similar to the set taken at the lower temperature is made. Heat is again applied to the oven and the process repeated.

Method of Determining the Number of Displaced Fringes.

First Method — by Direct Observation. — The displacement of the fringes means the number of dark bands and fractions of a band passing the reference circle on the under side of the cover-plate during any temperature interval. Upon heating the apparatus the expansion is generally so regular that by illuminating the field about every half minute the passage of the dark bands through the center of the reference circle can be followed by the eye and counted. If

the time of the passage of each dark band through the reference circle is recorded, there is little danger of mistake. In fact, having determined the period of the passage from the crossing of the first few bands, it is necessary only to look at the end of each period, so regular are the time intervals. Beside the number of whole bands counted during any temperature interval, $t_2 - t_1$, there are generally fractions of a band to be added or subtracted for each end of the interval, as the bands do not ordinarily come to rest in the center of the reference circle. These fractions are determined by the aid of the micrometer attachment. While the bands are stationary at the lower temperature, t_1, micrometer readings are taken on the positions of the five bands nearest the reference circle. These readings are denoted in the table as l_1, l_2, l_3, l_4, l_5 respectively, l_1 being the reading of the band located farthest to the left of the circle (Fig. 3). A reading is also taken upon the center of the reference circle. In the table this reading is denoted by l_0. The average of the readings on the five bands gives a very accurate reading for the position of the central band of the system, the one nearest the center of the reference circle. The difference between this average reading for the central band, and the reading on the circle, gives, in scale divisions, the distance the band is from the center of the reference circle. This distance OC divided by the width of a band gives the fraction by which the dark line is distant from the center of the reference circle. The width of a band is given by the expression $b = \frac{1}{6}[(l_5 + l_4) - (l_1 + l_2)]$. This fraction may have either a positive or negative sign and never exceeds $\frac{1}{2}$ in magnitude. If care is taken to mount the quartz apparatus so that, with increasing temperature, the fringes move in the same direction as the vertical cross-wires move when the micrometer is turned to the larger numbers, then the above method for determining the magnitude of the fraction will give also its proper sign. If the convention be adopted that the initial conditions are those at the

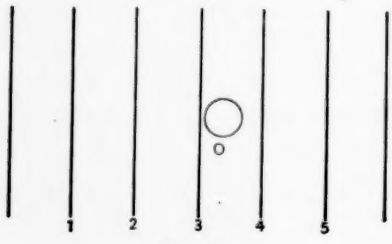

Fig. 3.

lower temperature, t_1, and the final conditions are those at the higher temperature, t_2, then the method works equally well for observations taken with rising or falling temperatures. Similarly when the second temperature limit, t_2, has been attained, and the fringes become stationary, the resulting fraction of a band with its proper sign is determined. The total displacement is given by the expression $f = M + \delta_{t_2} - \delta_{t_1}$ in which M represents the number of whole bands observed and δ_{t_2} and δ_{t_1} the fractions of a band at the temperatures t_2 and t_1 respectively.

Second Method — by Computations. — It is possible, as has been shown by Abbé,[1] to compute the number of displaced bands corresponding to any temperature interval, providing a second set of readings similar to those described in the first method is taken on fringes of another wave-length. As the green light of mercury vapor, $\lambda = 546\ \mu\mu$, is generally the most satisfactory to work with, it is always one of the two used. With quartz, when the air film is relatively thick, the only other color giving satisfactory fringes is the yellow due to mercury vapor, $\lambda_1 = 578.8\ \mu\mu$ and $\lambda_2 = 576.8\ \mu\mu$. This in reality gives two fringe systems which so completely overlap as to appear almost as a single system. With varying thickness of air film these two fringe systems exhibit the periodic variation in visibility first observed by Fizeau in the case of the sodium lines. At intervals of about 300 fringes, more accurately for each change in thickness of air film of 0.07924 mm., the combined fringe system exhibits maximum distinctness. In each of these regions of maximum distinctness there are about 100 bands of mean wave-length 577.8 $\mu\mu$ suitable for measurement. In the interval between these positions it becomes necessary to use the fringe system of one wave-length separately. The one corresponding to λ_2 has been used, and enough of this system appears on the edge toward the green to permit of accurate readings, although it is somewhat more difficult. Accordingly in the work with quartz three wave-lengths were used. That of the green is represented by λ_0, that of the average yellow, and the shorter yellow by λ as the case may be.

The change in the thickness of the air layer for a change in temperature, $t_2 - t_1$, may be represented by

[1] Wied. Ann., 38, 1889, p. 473.

$$[M_0 + \delta_{0t_2} - \delta_{0t_1}] \frac{\lambda_0}{2},$$

where the letters have the same significance as in the first method, the subscript o indicating that the green was used. Similarly

$$[M + \delta_{t_2} - \delta_{t_1}] \frac{\lambda}{2}$$

is an expression for the same change in thickness of the air film, when one of the yellow spectral colors is used. Placing these expressions equal,

$$[M_0 + \delta_{0t_2} - \delta_{0t_1}] \frac{\lambda_0}{2} = [M + \delta_{t_2} - \delta_{t_1}] \frac{\lambda}{2},$$

there is obtained

$$M_0 \frac{\lambda_0}{\lambda} = M + \delta_{t_2} - \delta_{t_1} - (\delta_{0t_2} - \delta_{0t_1}) \frac{\lambda_0}{\lambda}$$

or

$$M_0 \mu = M + \delta_{t_2} - \delta_{t_1} - (\delta_{0t_2} - \delta_{0t_1}) \mu \text{ where } \mu = \frac{\lambda_0}{\lambda}.$$

$$M_0 \mu = M + \rho \text{ where } \rho = \delta_{t_2} - \delta_{t_1} - (\delta_{0t_2} - \delta_{0t_1}) \mu.$$

That is to say, the number of whole green bands which have passed when reduced to an equivalent number of another color, will equal a number of bands plus a fraction of a band. A knowledge of the ratio between the wave-lengths of the colors used, λ_0/λ, enables one to compute the number of bands and the fraction of a band of the second color which corresponds to every whole number of green bands. In the following table, the columns headed M_0 contain the whole numbers of green bands. The maximum displacement of the green fringes for any temperature interval of 20° C. is not more than thirteen bands. The columns headed M and ρ contain respectively the corresponding whole number of yellow bands and fraction of a yellow band. It will be noticed that the fractions in the ρ columns do not repeat their values until eighteen green bands have been used. So, up to this point, each fraction is characteristic of a definite number of green bands. These fractions, of course, are all positive.

In the expression $M_0 \mu = M + \rho$, ρ is the fraction of a band of the

TABLE I.

Change in Thickness of Air Film in mm.	Green.	Yellow (Average) $\mu = 0.945$.		Yellow (Upper) $\mu = 0.9466$.		Change in Thickness of Air Film in mm.	Green.	Yellow (Average) $\mu = 0.945$.		Yellow (Upper) $\mu = 0.9466$.	
	M_0	M	ρ	M	ρ		M_0	M	ρ	M	ρ
0	0	0	0	0	0	0.0055	20	18	.90	18	.93
	1	0	.94	0	.94		21	19	.84	19	.88
	2	1	.89	1	.89		22	20	.79	20	.83
	3	2	.83	2	.84		23	21	.73	21	.77
	4	3	.78	3	.78		24	22	.68	22	.72
0.0014	5	4	.72	4	.73	0.0068	25	23	.62	23	.67
	6	5	.67	5	.68		26	24	.57	24	.61
	7	6	.61	6	.63		27	25	.51	25	.56
	8	7	.56	7	.57		28	26	.46	26	.50
	9	8	.50	8	.52		29	27	.40	27	.45
0.0027	10	9	.45	9	.47	0.0082	30	28	.35	28	.40
	11	10	.39	10	.41		31	29	.29	29	.34
	12	11	.34	11	.36		32	30	.24	30	.29
	13	12	.28	12	.31		33	31	.18	31	.24
	14	13	.23	13	.25		34	32	.13	32	.18
0.0040	15	14	.17	14	.20	0.0096	35	33	.07	33	.13
	16	15	.12	15	.15		36	34	.02	34	.08
	17	16	.06	16	.09		37	34	.96	35	.02
	18	17	.01	17	.04		38	35	.91	35	.97
	19	17	.95	17	.98		39	36	.85	36	.92

second color which has to be added to a whole number of bands of that color, M, in order to be equivalent to a whole number of green bands, M_0. ρ, therefore, must always have some one of the values found in columns ρ, as they are the only fractions which can be added to a whole number of yellow bands so as to give an equivalent number of green bands. ρ computed from $\rho = \delta_{t_2} - \delta_{t_1} - (\delta_{0t_2} - \delta_{0t_1})\mu$ will be found, therefore, in columns ρ of the table, and the whole number of bands of the two colors used, M_0 and M, are opposite the fraction. The two numbers M_0 and M, being known, the total expansion in green half-wave-lengths is $M_0 + \delta_{0t_2} - \delta_{0t_1}$. The same expansion in yellow half-wave-lengths is $M + \delta_{t_2} - \delta_{t_1}$. This latter value is reduced to the equivalent expression in green half-wave-lengths, and the average of the two expansions is considered the true expansion.

Occasionally the sign of ρ computed from the formula

$$\rho = \delta_{t_2} - \delta_{t_1} - (\delta_{0t_2} - \delta_{0t_1})\mu$$

is negative and as no negative fractions are found in the table, an equivalent positive fraction has to be found. If ρ is a negative quantity, $M_0\mu = M - \rho$. This may be written $M_0\mu = (M - 1) + (1 - \rho)$. The fraction $(1 - \rho)$ is now positive and may be found in the table. The number found opposite it in the column containing the whole number of yellow bands equals $M - 1$. M therefore, which is the quantity desired, equals this number plus 1.

CORRECTIONS FOR CHANGE IN REFRACTIVE INDEX OF AIR FILM.

As the index of refraction of the air layer between the two reflecting surfaces changes with a change of temperature, and also with an alteration of the atmospheric pressure, a suitable correction must be made upon the observed number of fringes displaced. With a rise in the temperature, the air becomes less dense, the wave-lengths of light used become correspondingly longer, and therefore fewer in number for a definite distance between the reflecting surfaces. The observed number of fringes is therefore too small, and this correction is always positive. Pulfrich[1] has shown that this correction may be computed by the formula

$$K_1 = d(t_2 - t_1)\frac{b_1}{760} \cdot \frac{1}{1 + \alpha t_1} \cdot \frac{1}{1 + \alpha t_2}\left[2\frac{(N - 1)\alpha}{\lambda}\right],$$

in which K_1 represents the correction, d the thickness of the air layer, b_1 the barometric pressure observed at the lower temperature, t_1, α the coefficient of expansion of air, and N its index of refraction under standard conditions.

The second correction, due to a change in atmospheric pressure, may evidently be positive or negative. If the pressure increases during a temperature interval, the corresponding increase in density of the air layer will result in shorter wave-lengths and more of them. Accordingly in this case the correction is negative, while a decrease in barometric pressure will give a positive correction. This is shown by Pulfrich to be

$$K_2 = d(b_2 - b_1)\frac{1}{1 + \alpha t_2}\left[2\frac{N - 1}{\lambda} \cdot \frac{1}{760}\right].$$

[1] Zeitschr. für Instr., 1893, p. 455.

The entire correction may therefore be represented by

$$K = K_1 + K_2 = d(t_2 - t_1)\frac{b_1}{760}\cdot\frac{1}{1+at_1}\cdot\frac{1}{1+at_2}\left[2\frac{(N-1)a}{\lambda}\right]$$
$$+ d(b_2 - b_1)\cdot\frac{1}{1+at_2}\left[2\frac{N-1}{\lambda}\cdot\frac{1}{760}\right].$$

These corrections amount to a considerable fraction of a wave-length when the thickness of the air layer is large.

The Degree of Accuracy Attainable.

The formula expressing the value of the coefficient of expansion

$$a = \frac{f_0\cdot\frac{\lambda_0}{2}}{L(t_2 - t_1)}$$

shows that the value of a depends upon four measurable quantities, f_0, the number of displaced fringes, L, the length of the object whose expansion is to be measured, and the two temperatures, t_2 and t_1.

In the expression

$$f_0 = M_0 + \delta_{0t_2} - \delta_{0t_1},$$

the δ's can be determined with an average error of $\pm$ 0.015 and f therefore with a mean error of $\pm$ 0.02. The error in the value of a resulting from this error is $\pm$ 3.5 $\times$ 10^{-8}. The effect of the errors of observation in the measurement of L and the t's upon a will vary with the magnitude of f_0 if the temperature intervals $t_2 - t_1$, be assumed a constant, as 20° C. At room temperatures f_0 is approximately 4.3 and at 500° C., 13.3. The error in the measurement of L is $\pm$ 0.001 mm. and the corresponding error in a at room temperature is .1 $\times$ 10^{-8} and at 500° C. is .3 $\times$ 10^{-8}. At the lower temperatures where the thermometers graduated in fifths are used, the difference in the temperatures, $t_2 - t_1$, can be determined with an error of $\pm$ 0°.05 resulting in an error in a of 1.9 $\times$ 10^{-8}. When the thermometers graduated in degrees are used, the interval, $t_2 - t_1$, can be determined with a mean error of $\pm$ 0.2°. This results in an error in a of 23 $\times$ 10^{-8} at the upper temperature limit of 500° C.

With the platinum resistance thermometer, to be described later, the temperature intervals can be determined throughout the entire temperature range of 500° C., with a mean error of ± 0.05°. This gives an error in α at 500° C. of 6. × 10^{-8}. The total effect of the errors of observation upon the result is therefore $E = \sqrt{E_f^2 + E_L^2 + E_t^2}$. At room temperature this amounts to 3.9 × 10^{-8}. At 500° C. with the mercury thermometers the total error is 23.2 × 10^{-8}, while with the platinum resistance thermometer this is reduced to 6.9 × 10^{-8}.

EXAMPLE OF A COMPLETE OBSERVATION AND COMPUTATION.

Length of quartz ring, 9.8426 mm.
Length of quartz cylinder, 2.0900 mm.
Effective length of quartz expanding, 7.7526 mm.
Thickness of air layer, 7.7526 mm.

$t_1 = 489.5°$ C. $b_1 = 740.0$ mm.

	Green.		Yellow (Upper).		
l_1	74	76	l_1	19	20
l_2	172	171	l_2	119	118
l_3	264	261	l_0	183	185
l_0	284	284	l_3	218	216
l_4	351	352	l_4	309	308
l_5	442	440	l_5	406	404
l_c	260.6	260.0	l_c	214.2	213.2
OC	−23.4	−24.0	OC	+31.2	+28.2
b	91.1	90.8	b	96.1	95.7
δ	−.256	−.264	δ	+.325	+.295
Ave. δ	−.260			+.310	

$t_2 = 509.6°$ $b_2 = 741.0$ mm.

	Green.		Yellow (Upper).		
l_1	90	94	l_1	69	67
l_2	181	185	l_2	167	167
l_0	275	277	l_3	260	258
l_3	279	280	l_0	275	276
l_4	363	366	l_4	354	356
l_5	447	450	l_5	449	450
l_c	272.0	275.0	l_c	259.8	259.6
OC	−3.0	−2.0	OC	−15.2	−16.4
b	89.8	89.5	b	94.5	95.3
δ	−.033	−.022	δ	−.161	−.172
Ave. δ	−.027		Ave. δ	−.166	

$$\delta_{0t_2} = -.027 \quad \delta_{0t_1} = -.260 \quad \delta_{t_2} = -.166 \quad \delta_{t_1} = +.310$$

$$t_2 - t_1 = 20.1^\circ \quad \frac{t_2 + t_1}{2} = 499.5^\circ \quad b_2 - b_1 = +1.0 \text{ mm.}$$

$$\rho = (\delta_{t_2} - \delta_{t_1}) - (\delta_{0t_2} - \delta_{0t_1})\mu = -.696$$

$$\rho + 1 = +.304 \text{ (Table .31)} \quad M_0 = 13 \quad M - 1 = 12 \quad M = 13$$

$$f_0 = M_0 + \delta_{0t_2} - \delta_{0t_1} = 13.233 \quad f = M + \delta_{t_2} - \delta_{t_1} = 12.524$$

$$\text{Ave. } f_0 = \left\{ \begin{array}{r} 12.524 \\ \overline{.9466} = 13.231 \\ 13.233 \end{array} \right\} \cdot 13.232.$$

Correction to f_0 due to change in index of refraction of air.

$K_1 = +.075 \quad K_2 = +.004 \quad K = +.079 \quad f_0' = f_0 + K = 13.311.$

$$\alpha = \frac{f_0' \cdot \frac{\lambda_0}{2}}{L(t_2 - t_1)} = 2332 \times 10^{-8} \text{ at } 499.5^\circ.$$

Results of Observations.

The results obtained at this time are given in Table II. The fourth column contains the value of α derived from the observations multiplied by 10^8. These results are represented in the plot, Fig. 5, by the circles.

Second Part.

As was mentioned in the introduction, certain considerations seemed to render desirable a repetition of the work just described. First no data was obtained in the temperature interval 350° to 420° C. as 420° were the lowest reading on the fourth thermometer appearing above the insulation of the oven. Second, as shown earlier, the error in measuring the temperature interval may be large in comparison with the error in the other measurements. This error is inherent in the use of the mercury thermometer, and combines the error arising from the constantly changing zero point, with those due to scale reading, and to uncertainties of stem and calibration corrections. Third, there may be an actual difference in temperature between the quartz apparatus and the thermometer

TABLE II.

t_1	t_2	$\frac{t_1+t_2}{2}$	$\alpha \times 10^5$
152.9	168.2	160.5	959.4
178.4	196.4	187.4	1,008
178.4	213.9	196.1	1,024
196.4	213.9	205.1	1,044
223.1	257.7	240.4	1,092
249.5	272.9	261.2	1,138
257.8	282.8	270.2	1,151
272.9	290.9	281.9	1,182
282.8	302.1	292.4	1,218
285.7	306.4	296.0	1,227
285.7	313.8	299.7	1,247
290.9	313.9	302.4	1,247
302.1	319.8	310.9	1,281
306.4	327.5	316.9	1,289
306.3	327.6	316.9	1,294
319.8	336.6	328.2	1,339
329.8	347.0	338.4	1,363
330.9	345.3	338.1	1,368
327.5	345.3	336.4	1,355
331.2	356.2	343.7	1,381
343.2	356.2	349.7	1,400
409.3	431.3	420.3	1,643
416.4	431.3	423.9	1,689
416.4	438.5	427.5	1,690
423.5	446.3	434.9	1,722
423.5	450.0	436.7	1,746
438.5	460.9	449.7	1,805
449.1	471.2	460.1	1,878
450.9	467.2	459.1	1,885
449.1	482.2	465.7	1,901
460.9	482.2	471.6	1,934
464.9	489.4	477.2	1,932
477.1	499.2	488.2	2,168
489.5	499.2	494.3	2,186
489.4	509.6	499.5	2,331
489.5	509.6	499.6	2,332
499.2	509.6	504.4	2,469

bulb as the former is within a brass chamber at the end of a tube extending out of the oven and the latter outside the chamber at a distance of 1.5 to 2.5 cm.

Changes in Apparatus.

In the redetermination a platinum resistance thermometer was substituted for the mercury thermometers and an entirely different method adopted for supporting the quartz apparatus within the oven.

As the laboratory possessed a large Post Office box made by Nalder Bros. which was known to be in excellent adjustment, the platinum thermometer constructed was one adapted to that method of measuring resistance. The terminals of the fine platinum wire, wound on the usual rectangular mica frame, were gold soldered to heavy double platinum leads. These leads, insulated in the stem of the thermometer by mica disks, were soldered externally to heavy copper leads terminating in mercury cups, so arranged that either pair of leads separately, or the leads together with the fine coil, could be put in the arm of the box for measurement by connecting the proper cups by means of heavy copper links. This thermometer was calibrated as recommended by Callendar and Griffiths,[1] and was capable, together with its accessory electrical apparatus, of measuring a temperature variation of 0.01°. In a determination, the resistance of the thermometer leads was first measured, then the resistance of the entire thermometer, followed by the interferometer readings. The resistance of the entire thermometer was again determined, and then the resistance of the leads. The temperatures thus obtained before and after the interferometer readings, which consumed about fifteen minutes, seldom varied by more than 0.02° or 0.03° C.

The arrangement for mounting the quartz apparatus is shown in Fig. 4. It consists of a cylindrical brass chamber, firmly supported by two solid porcelain rods 50 cm. long and 8 mm. in diameter, clamped at their upper ends to an iron bar, this bar being supported by clamps and rods to the pier. The wall of this chamber is formed by a solid brass cylinder, 6 cm. long, 7.7 cm. in diameter and 3 mm. thick, which fits tightly into a groove turned into the brass plate serving as a bottom. The cover, by means of a similar groove fits tightly on the cylinder. This cover plate is furnished with three circular apertures, the central one being fitted tightly with a thick

[1] Phil. Trans. Roy. Soc., 1891, A, p. 141.

glass disk, serves as a window for admitting the beam of light, and the two on opposite sides of this are for the introduction of the thermometers.

To mount the quartz within the chamber, the thermometers are first run through the holes in the cover, and then the brass cylinder and cover are lifted to a sufficient height and held by any convenient support. The quartz is then mounted on the brass disk supported by the leveling screws through the base. Two thin brass rods (not shown in figure) are screwed firmly to the bottom of the chamber near the edge. These rods are bent at right angles and so adjusted that the thermometers fastened to them just escape contact with the quartz. When the wall and top of this chamber are lowered into place, the quartz and the thermometers are completely surrounded by the brass chamber with the single exception of the thick glass window at the top. This chamber is in turn completely surrounded by the air of the oven. Thus the chimney-like effect due to the porcelain tube of the earlier arrangement is avoided, as is also the questionable arrangement of the large window with the thermometer bulb outside. As it is possible to maintain the temperature of the air of the oven surrounding the brass chamber constant, to within a tenth of a degree, for intervals of half an hour to an hour, it would seem that any possible difference in temperature between the quartz and the thermometers due to a different rate of following temperature changes, would be extremely small. To introduce the beam of light into the oven, a tube, provided with three glass plates to prevent air currents, is mounted in the insulation directly above the center of the brass chamber.

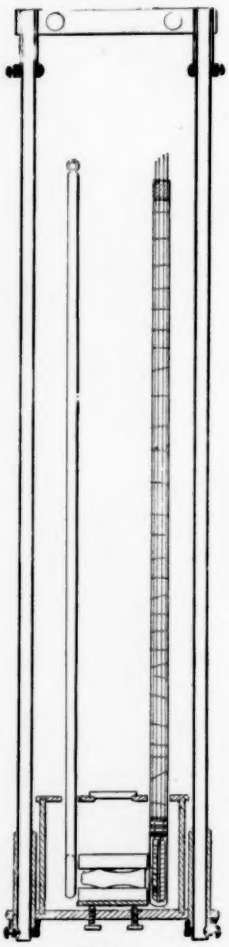

Fig. 4.

As an additional protection for the interferometer against heating effects from the oven, two asbestos shields were mounted between the instrument and the oven. A thermometer, in contact with the interferometer tube directly over the oven, was read with each set of observations. The results show that the temperature of the interferometer varied from a minimum of 1.2° C. to a maximum of 2.0° C., above that of the room while the room temperature varied during the entire course of the investigation 2.7° C. It may be concluded therefore, that the heat from the oven does not produce measurable disturbing effects.

Results of Later Observations.

Two series of measurements were made with the apparatus just described. One of them comprised a few determinations between room temperature and 100° C., in which both platinum resis-

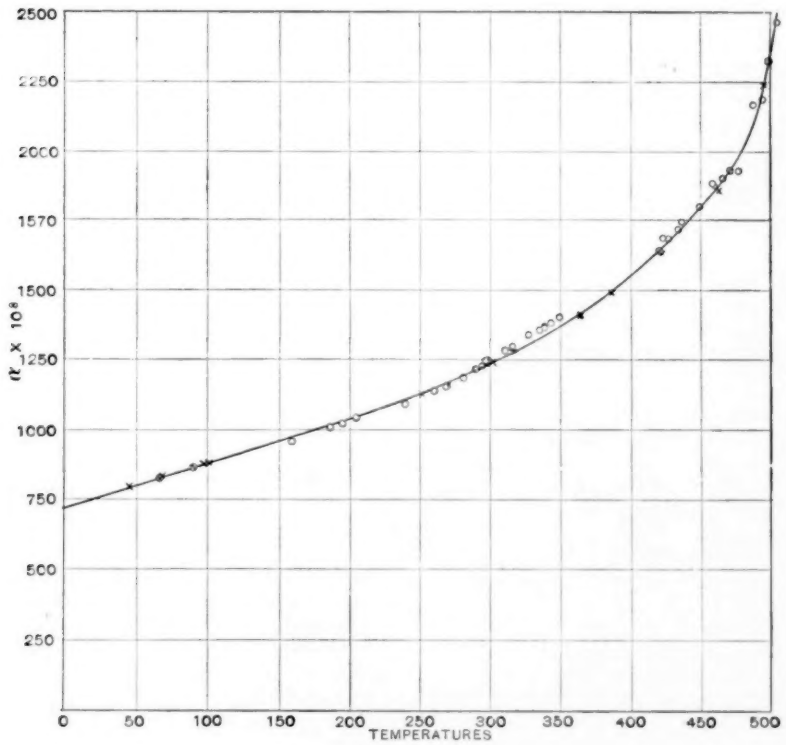

Fig. 5.

tance thermometer and standard mercury thermometer were employed. The results obtained are contained in Table III. and are shown in the plot by the combined cross and circle. The two thermometers agreed to the limits within which the mercury thermometer could be read. A more extended set of results was obtained extending to 500° C., in which the resistance thermometer only was used. The results are contained in Table IV., and are indicated in the plot by the crosses. The results obtained in 1901, as shown by the circles, are in substantial agreement with the recent redetermination, the region between 300° C. and 350° C. showing the only marked difference. Here the results obtained by the mercury thermometer are perceptibly higher. As the thermometer used in this region had unfortunately been broken, no opportunity was offered for discovering the cause of this difference.

TABLE III.

t_1	t_2	$\frac{t_1+t_2}{2}$	$\alpha \times 10^8$	$\alpha' \times 10^8$	$(\alpha-\alpha') \times 10^8$
56.54	78.56	67.55	825.8	826.4	+0.6
78.56	103.73	91.15	864.2	864.6	+0.4

TABLE IV.

36.24	51.98	44.11	790.0	788.5	− 1.5
51.98	85.35	68.66	830.6	828.3	− 2.3
21.44	173.23	97.83	875.6	875.4	− .2
85.35	114.81	100.08	877.8	880.3	+ 2.5
241.78	260.62	251.20	1129.5	1127.8	− 1.7
260.62	279.88	270.25	1156	1161	+ 5.0
279.88	328.62	304.25	1244	1233	−11.0
328.62	401.20	364.91	1408	1409	+ 1.0
328.62	399.43	364.02	1406	1407	+ 1.0
328.62	442.50	385.56	1490	1486	− 4.0
399.43	442.50	420.97	1639	1639	0.0
446.67	480.01	463.34	1862	1865	+ 3.0
487.39	504.58	495.98	2241		

Up to 250° C. the results may be represented by the equation $\alpha_t = (717.0 + 1.620t) \times 10^{-8}$. Between 250° and 470° C. the expression $\alpha_t = [1125.0 + 1.65(t - 250) + 0.00566(t - 250)^2 + 0.0000134(t - 250)^3] \times 10^{-8}$ represents the results quite closely.

The sharp upward turn of the curve in the neighborhood of 500° C. renders it impracticable to represent the results further by an equation. The values of α_t derived by means of these equations are contained in the column headed $\alpha' \times 10^8$ in the tables III. and IV. The last column of these tables contains the differences between α and α' in units of the eighth decimal place, and indicates how closely the equations may be expected to represent the results determined by direct measurement.

COMPARISON OF RESULTS WITH THOSE OF OTHER OBSERVERS.

The following table contains the results obtained in the investigations mentioned in the introduction.

$$\text{Fizeau } \alpha_t = 10^{-8}(710 + 1.770t).$$
$$\text{Benoit } \alpha_t = 10^{-8}(716.1 + 1.602t).$$
$$\text{Reimerdes } \alpha_t = 10^{-8}(692.5 + 1.689t).$$
$$\text{Scheel } \alpha_t = 10^{-8}(714.4 + 1.630t).$$

while from my observations for the interval 0° to 250° we have $\alpha_t = 10^{-8}(717.0 + 1.620t)$.

A comparison of these expressions shows that the results obtained in this investigation agree very closely with those of Benoit and Scheel. If the expansion for 1 meter be computed for intervals of temperature of 50° and 100°, the following values are obtained.

Benoit	(0°–50°)	378.1 μ	(0°–100°)	796.2 μ
Scheel		377.6		795.9
Randall		378.7		798.0

The differences in each case lie well within the limits of the experimental error.

As far as is known no accurate determinations of the coefficient of expansion of quartz beyond 220° have previously been made. Mallard and Le Chatelier[1] have made a rough determination of the expansion of quartz cut parallel to the principal axis up to 1,000° C. By means of two cameras properly mounted they were able to photograph the ends of a quartz rod 100 mm. long, both before and after it had been heated through a definite range of temperature. As the plates were ruled in $\frac{1}{10}$ mm. divisions, the expansions could

[1] Comp. Rendus, No. 108, p. 1046.

be read off directly from the plates, hundredths of a mm. being estimated with some degree of accuracy. Their results are given in the following table:

Temperature.	First Determination.	Second Determination.
15°	0	0
270°	0.20 mm.	0
480°	0.53 "	0.55
570°	0.93 "	0.93
660°	0.95 "	0.99
750°	0.95 "	0.95
910°	0.90 "	0.87
990°	0.90 "	0.86
1060°	0.90 "	0.89

The numbers in columns 2 and 3 are the expansions expressed in mm. when the change of temperature is from 15° C. to the temperature opposite a given expansion. The table shows that a great change in the rate of expansion occurs between the temperatures 480° C. and 570° C. Le Chatelier calls attention to this in the following statement: "These measurements on the expansion of quartz seem to indicate a very rapid transformation in the properties of quartz without proving it in an indisputable manner. We may also represent the results of the experiment by assuming between 400° and 600° C., a very rapid but continuous increase in the expansion which leads to a curve presenting a point of inflection, the tangent at which will be sensibly vertical, a very improbable hypothesis, it is true. It would be interesting to verify this point of sudden transformation by the study of phenomena subject to more accurate measurement than that of expansion by heat. At the same temperature, 570° C., the circular polarization, the double refraction, the electric conductivity, the specific heat of quartz should experience equally rapid changes."

This agrees well with the rather sudden rise in value of the coefficient after 480° C., as shown by the upward turn of the plotted curve, and if the expansions of a rod of quartz 100 mm. long be computed for the temperature ranges 15° — 270° and 15° — 480° by means of the two equations resulting from this investigation, they are found to be .24 mm. and .55 mm. respectively, agreeing well with those obtained by Le Chatelier. Sahmen and Tammann,[1]

[1] Ann. der Physik und Chemie, 10, 1903, p. 879.

working with two samples of quartz, find expansions for these intervals of approximately .22 mm. and .61 mm. and .26 mm. and .89 mm. respectively, results differing from each other by more than observational error and both considerably larger at the higher temperature than the results of Le Chatelier, or those of the author.

To Professor Reed, under whose immediate direction this investigation was conducted, the writer wishes to express his grateful thanks for the interest shown and the many valuable suggestions offered during the progress of the work.

PHYSICAL LABORATORY, UNIVERSITY OF MICHIGAN,
ANN ARBOR, September 9, 1904.

COEFFICIENTS OF LINEAR EXPANSION AT LOW TEMPERATURES.

By H. D. Ayres.

IN passing from ordinary temperatures upward it is a well-known fact that there is a slight increase in the coefficient of expansion of many substances. It has been thought probable that the curve thus obtained will hold for lower temperatures. But little is known as to the behavior of bodies at extremely low temperatures. Almost the only work that has been done at temperatures below — 50° C. is that of Dewar,[1] and of a Russian physicist, Zakrzewski.[2] In the work of the former the specific-gravity method was used in studying the behavior of bodies such as ice, some hydrated salts, solid carbonic acid, etc. Metal balls were used in the determination of the specific gravity, and the work was based on the assumption that these metals contract according to the Fizeau formula for coefficient of expansion, it being assumed that the parabolic formula might be legitimately extended to low temperatures. Zakrzewski examined the behavior of glass, iron and copper to — 100° C., finding the coefficients to decrease somewhat rapidly.

The ordinary methods for the determination of the expansion of bodies become difficult of application at low temperatures, for the reason that mirrors, lenses, etc., either attached directly to, or anywhere near a body having a temperature nearly two hundred degrees below zero centigrade, very quickly become frosted. Another fact to consider is that the body must be quite small if it is to be immersed in liquid air and kept at nearly a constant temperature for any length of time: that is to say, the liquid air must be kept in a receptacle which will prevent its rapid evaporation. Another necessary condition is that the body itself be well insulated, else the heat from the outside is conducted to it so rapidly

[1] "Expansion of Solids at Low Temperatures," Royal Soc. Proc., 70, July 8, 1902, pp. 237–246; Nature, Vol. 66, May 22, 1902, p. 88.

[2] "Über die Ausdehnung einiger fester Körper bei niederen Temperaturen," Beibl., 1890, p. 491.

that it does not readily come to a temperature of equilibrium, and the parts expand and contract unequally. Until the behavior of at least one substance is known it is certain that no method of compensation can be used.

The principle used in the work now to be described is that first used by Fizeau [1] in the determination of the coefficient of dilatation of crystalline substances. The method devised by him depends essentially upon the determination of the difference of expansion between the screws of a small metallic tripod and the object under investigation, which is supported by it. The method has been employed with some modifications by Pulfrich,[2] and with still other modifications, including a method of compensation, by Tutton.[3]

The apparatus used in the present work, however, is on an entirely different plan, and is much simpler. The dilatometer as described by Pulfrich and Tutton, has been replaced by a single piece of metal[4] — the metal under consideration — used between two glass plates, thus very much simplifying the work in some ways, though it must be acknowledged, complicating it in others. It is believed, however, that the sources of errors are at least considerably decreased, since the expansion of the tripod screws and the need of compensation are eliminated. The metal piece is cut into the form of a cylindrical shell with its faces nearly parallel, so that the interference phenomena take places between the light reflected from the upper surface of the lower glass plate and the lower surface of the cover plate. The lower plate has one plane surface and is of black astronomical glass so that all light is absorbed except that reflected from the plane surface. The cover plate has both surfaces plane and inclined at a small angle, preferably about 30′, though one or two were used in the course of this work with an angle of three or four degrees. By having the faces inclined the reflection from the upper surface is thrown out of the field of vision and does not illuminate the dark spaces between the bands.

As a mark by which to count the bands as they pass, a small

[1] Compt. Rendus, Vol. 58, p. 923, and Vol. 62, p. 1133. Ann. Chim. Phys. (4), Vol. 2, p. 143; (4), Vol. 8, p. 333.

[2] Zeitschrift für Instrumentenkunde, 1893, p. 365.

[3] Phil. Trans., Vol. 191, A, 1899, p. 313.

[4] Annalen der Physik. (9), Vol. 314, 1902, p. 837.

silver disc about a millimeter in diameter on the lower surface of the cover glass is used. This may be easily obtained by silvering the whole surface and then fastening the plate to a wooden block and revolving it in a small lathe, cleaning the surface, except this spot, by means of a small piece of soft wood. This is wet so as to avoid scratching the glass surface. In the case of one of the plates used a small hole was drilled through its center. The plates are circular in shape, 30 mm. in diameter and about 2 mm. in thickness. They are held firmly in contact with the metal by means of a small cylindrical lead weight having an opening through its center and resting on the upper cover plate, (Fig. 1, *a*). In order to decrease the likelihood of disturbances due to particles of dust or dirt between the faces of the metal and the glass surfaces, the faces of the metal are cut so as to leave three equidistant projections upon which it may rest (Fig. 1, *I*). The length of the metal cylinder used depends altogether upon the light available. If sodium light is used, the thickness must be very small, something like one or two millimeters, and the piece so ground that this thickness gives a maximum visibility for the interference bands. If the green mercury line is employed an air space of over a centimeter may be used.

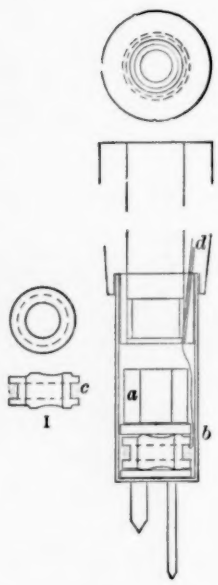

Fig. 1.

The metal piece, plates, and lead weight are placed in position in a small cylindrical brass cup, something over 3 cm. in diameter and about 8 cm. in length, Fig. 1, *b*. This is easily accomplished by adjusting the pieces properly in an inverted position upon a support which can enter then brass cup, then placing the cup in position and inverting the whole. The brass cup fits closely over the end of a wooden tube, and is held by friction. The wooden tube is so turned that it slips into the brass cup for some distance and has in addition a flange, which extends for a short distance over the outside of the cup. This joint is liquid-tight and its object is two-fold: First to facilitate the adjustment of the different parts and of

the lead-wires to the coil used for temperature measurement, and second to act as a non-conductor of heat when the whole end is immersed in liquid air. The metal cup has soldered to its bottom and projecting downwards two metal pieces, one about 2 cm. in length and quite thick, the other one about twice as long but smaller in diameter. Both of these have their ends pointed, and their purpose is to extract the heat from the interference apparatus and brass cup slowly, which could not be done if the liquid air came in contact with the bottom of the cup before it was well cooled. The wooden tube is about 30 cm. long and its upper end fits into a wooden support which clamps it firmly, thus leaving the lower end free, so that it can extend down into a vessel containing water or a flask of liquid air, etc. To alter the temperature the receptacle containing the liquid is raised, the liquid rising around the brass cup and wooden tube. The movement must be very slow and under control so that the bands may be made to shift at such a rate that they may be counted. A piece of apparatus was made for this purpose consisting of a large, heavy tripod stand and to its vertical rod another smaller rod was attached at a distance of about 15 cm. Then a stage was made to move up or down these rods, being supported from above by a leather strap which passed to a wheel and axle. The axle worked with a friction grip, thus permitting the stage to move as slowly or rapidly as desired. For extremely slow movement a wooden arm can be attached to the wheel.

Illuminating and Observing Apparatus.

The illumination is that from a Cooper-Hewitt mercury arc lamp, the green line being used. This is obtained by separating the colors by a prism, *P*, Fig. 2, the light first being passed through the lens *A* to render it parallel. Beyond the prism is another lens, *B*, whose principal focus is at p. A small total reflecting prism is situated at p, which turns the light at right angles in a horizontal plane. It is again turned at right angles, downward, by the mirror *M*, which is silvered on its reflecting face. The mirror was found to serve much better in this position than a total reflecting prism, being much more easily adjusted. At *D* is placed an achromatic lens of 25 cm. focal length, and so adjusted with respect to p as

to act as a collimater, thus rendering the rays passing to the interference apparatus parallel. From the interference apparatus the light is reflected back along the same path and brought to a focus at p, forming a real image of the rectangular aperture of the total reflecting prism. Adjustment is so made that this is formed just to the right and touching the edge of the prism. The interference pattern would be observed at the conjugate focus, from the interference apparatus, of the lens D. In order to be nearer the apparatus for adjustment, etc., and also to gain in intensity of light,

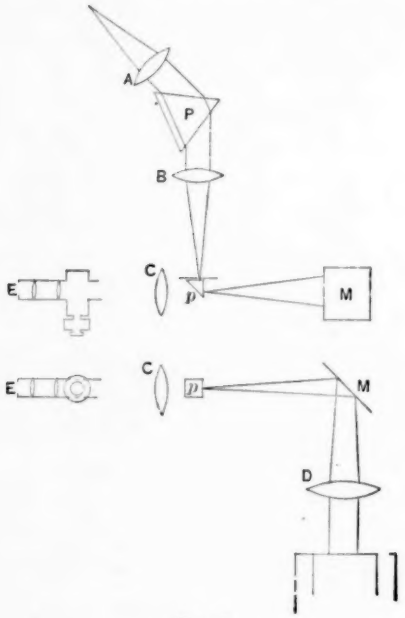

Fig 2.

another lens, C, of about 20 cm. focal length, is placed just back of p so that the rays are focused about six or eight centimeters to the left of p. For the same reason the mirror M is brought between D and p instead of the light passing through the lens D before reflection. To observe the interference pattern a Ramsden double eyepiece, E, is employed, which focuses the double cross hairs as in Tutton's apparatus, the lines being moved, and not the eyepiece itself, with respect to the interference bands and the silver disc. The eyepiece also contains fixed cross hairs. It is mounted upon

one of the posts of an optical bench, and thus has a rack and pinion vertical adjustment and a micrometer screw horizontal adjustment. The lens C is also mounted so as to be adjustable in any direction, as are also the mirror M and the lens D. The prism p is mounted on another post of the bench, and is fitted in a brass case, having on the side towards B an adjustable rectangular opening which can be varied in size from the size of the face of the prism to as small an opening as is desired. A large opening is necessary in making adjustments, but to obtain clear bands with a path difference of several millimeters the aperture must be cut down very much; one or two square millimeters give the best results in most cases.

Temperature Measurement.

The measurement of temperature is made by means of the change in resistance of a coil of very fine copper wire wound closely round the metal cylinder (see Fig. 1, *c*.) The ends of this fine wire are connected to larger wires which serve as lead-wires, and these latter pass out through small holes bored in the wooden tube, *d*. A calibration curve was obtained by finding the resistance of the coil at four points, namely, about 100°, 0°, − 80°, the latter by immersing the coil in solid carbon dioxide mixed with ether, and at about − 190° by immersing in liquid air. The temperature of the air was obtained from Baly's[1] curve. The air given off by the liquid while the coil was immersed is analyzed and the percentage of oxygen found.

A Wheatstone's bridge was used in determining the resistances of the coil at the different temperatures. The deflections of the galvanometer were observed by means of a spot of light reflected on a ground-glass scale, which was set directly in front of the expansion apparatus and directly in line with EM, Fig. 2, so that the attention could be readily and quickly turned from the spot of light to the eye-piece, or *vice versa*.

Adjustment and Use of the Apparatus.

In adjusting the apparatus the eye-piece E, Fig. 2, and the lenses C and D are removed and the tube is adjusted vertically so that the light passes through it parallel to its length. This is most easily

[1] Phil. Mag. (5), 49, 1900, p. 517.

accomplished by using daylight and removing the metal cup from the end of the tube. The lens D is placed in position and the distance CMD made about 25 cm. The metal cup, with a single reflecting surface in it, is then attached. This reflecting surface should be at about the same position as the lower surface of the cover plate when it is finally placed. The arc is now started and the tube so adjusted, by means of the set-screws against its support, that the light is reflected back to p. An ordinary eye-piece is adjusted so that it focuses a point just to the right of the edge of the prism. The lens D is then moved until an image of the rectangular opening at p, which must be quite small, is clearly in focus. Then replacing the eye-piece by the lens C and the Ramsden eye-piece E, and placing in position the two plates and metal instead of the single reflecting surface, the interference bands should appear in the field. The width of the bands is now measured by the movable double lines, and the initial position of the band nearest the reference mark found. This distance, divided by the width of the bands, gives a fraction to be added to the whole number of bands counted, if the bands move so that the band nearest the reference mark moves towards the mark, or one minus the fraction added if the bands move in the opposite direction.

Owing to the condensation and freezing of moisture on the plates some drying reagent must be introduced in the wooden tube after all adjustments are made, and allowed to remain until the air is completely dry. Upon removing this the tube is closed by means of a plane cover glass, the end of the tube being cut so that the cover glass is inclined at a small angle to a line at right angles to the length of the tube. It is then made air-tight by sealing with shellac. Through the wall of the wooden tube near its upper end is a small metal tube, by means of which connection is made to a drying tube, and as the air inclosed in the brass cup and wooden tube cools and decreases in volume, dry air enters.

Before beginning a set of observations the resistances of the coil for the desired temperatures were obtained from a calibration curve, and the box resistance adjusted for the first temperature. The liquid air was now raised slowly and the bands began to move. A key in the galvanometer circuit was closed frequently to note

the deflection. A D'Arsonval galvanometer was used and two pins stuck in the woodwork back of the air vane, one at each end, prevented more than a very small deflection of the needle. When the deflections had become less violent the circuit was left closed, and soon the spot of light began to move towards the zero position. The cooling at this point must proceed very slowly, so that the metal and coil may come to the same temperature, and it is best that the cooling cease entirely just about the time the spot of light reaches its zero position. By proceeding slowly enough this can be obtained almost exactly. In any case, however, the spot of light can be shifted to the one side or the other of the zero mark by slightly raising or lowering the liquid, the bands meanwhile moving a small fraction of the distance between two bands, one way or the other, thus proving very conclusively that coil and metal are at the same temperature. The cross hairs may easily be set just as the spot of light is on the zero position. The fraction of the band is thus obtained, and together with the fraction found in starting, added to the whole number of bands counted. This gives the expansion of the body in terms of the wave-length of light used. This number of bands is different, however, from what it would have been if the refractive index of the air inclosed in the metal cylinder had not changed, due to change in temperature. The formula for making this correction has been worked out and explained by Pulfrich in the article referred to above. He gives in addition to this correction one for change in barometric pressure, but this is always small, and since in this work a temperature interval is covered in about one hour the small change in pressure likely to occur may be entirely ignored. We shall consider therefore only that part due to change of temperature. Since the change in length of the metal spool due to change in temperature is slight, we can assume the thickness of the air space as constant in the correction. Call this d. Then

$$d = {}_1\beta_1 \frac{\lambda_1}{2}$$

where λ_1 is the wave-length of light and ${}_1\beta_1$ the number of wave-lengths in the path difference of the light reflected from the two surfaces. Now suppose the above temperature change to have

taken place without affecting the length of the metal spool. A certain number of bands would have shifted, caused by the change in wave-length due to change in refractive index. Denoting this new wave-length by λ_2 and the corresponding number of wave-lengths by β_2, again we have

$$d = \beta_2 \frac{\lambda_2}{2}.$$

Since the correction, k, is equal to $\beta_1 - \beta_2$ (supposing the temperature to increase) we obtain from the above and the relation

$$\lambda_2 = \frac{\lambda_1 \mu_1}{\mu_2},$$

the expression,

$$k = \frac{2d}{\lambda_1} \cdot \frac{\mu_1 - \mu_2}{\mu_1}.$$

As λ_1 differs a very little from λ, and μ_1 is very nearly equal to *one*, the expression may be written,

$$k = \frac{2d}{\lambda} (\mu_1 - \mu_2). \tag{1}$$

By Gladstone and Dale's law,

$$\frac{\mu - 1}{\rho} = \text{constant}$$

where μ is the refractive index at any temperature and ρ the density. Then,

$$\mu_s - 1 = K\rho_s, \tag{2}$$

the subscript s denoting standard conditions. From this we have,

$$\mu_{t_1 b} - 1 = K\rho_s \frac{b}{760} \cdot \frac{1}{1 + \alpha t_1} \tag{3}$$

in which the temperature is t_1 and the barometric pressure b, and α the coefficient of expansion of air. Replacing $K\rho_s$ by its value from (2),

$$\mu_{t_1 b} - 1 = (\mu_s - 1) \frac{b}{760} \cdot \frac{1}{1 + \alpha t_1}. \tag{4}$$

For a temperature t_2 this becomes,

$$\mu_{t_2 b} - 1 = (\mu_s - 1) \frac{b}{760} \cdot \frac{1}{1 + \alpha t_2}. \tag{5}$$

Subtracting (5) from (4), and substituting in (1) where the subscript 1 corresponds to $t_1 b$, and 2 to $t_2 b$, we obtain,

$$k = \frac{2d}{\lambda}(\mu - 1)\frac{b}{760} \cdot \frac{\alpha(t_2 - t_1)}{(1 + \alpha t_1)(1 + \alpha t_2)},$$

the subscript s being dropped. Then if f denote the number of bands counted, f' the corrected number, $f' = f + k$ or

$$f' = f + d(t_2 - t_1) \cdot \frac{b}{760} \cdot \frac{1}{1 + \alpha t_1} \cdot \frac{1}{1 + \alpha t_2} \cdot \frac{2(\mu - 1)\alpha}{\lambda}.$$

The logarithm of $[2(\mu - 1)\alpha]/\lambda$, as given by Pulfrich for the green mercury line, $\lambda = 0.0000546$ cm. is $7.59901 - 10$.

The coefficient of expansion e is given by the expression,

$$e = f' \frac{\lambda(t_2 - t_1)}{2d}. \qquad (6)$$

Following are some results for aluminium:

SERIES I. *Aluminium.*

d = length of metal spool = 0.603 *cm.* *b = barometric pressure* = 74.0 *cm.*
λ = 0.0000546 *cm.*, $\lambda/2$ = 0.0000273 *cm.*

Temperature Interval.	Bands Counted.	Correction.	Total.	$e \times 10^{-7}$
0 to − 40	17.7	1.1	18.8	213
− 40 " − 80	15.8	1.5	17.3	196
− 80 " −120	13.9	2.3	16.3	184
−120 " −160	9.5	4.0	13.5	152
−160 " −185	3.6	4.0	7.7	139
0 " 40	18.0	0.8	18.8	213
40 " 78	19.2	0.6	19.8	236

SERIES II. *Aluminium.*

d = 0.603 *cm.* *b* = 74.3 *cm.*

Temperature Interval.	Bands Counted.	Correction.	Total.	$e \times 10^{-7}$
0 to − 40	17.7			
− 40 " − 80	16.0			
− 80 " −120	13.9			
−120 " −160	9.9			
−160 " −175	2.0			
−175 " −160	2.0	2.4	4.4	133
−160 " −120	10.0	4.1	14.1	159
−120 " − 80	14.0	2.3	16.3	185
− 80 " − 40	16.1	1.5	17.6	199
− 40 " 0	17.8	1.1	18.8	213
0 " 40	18.4	0.8	20.2	229
40 " 80	19.6	0.7	20.3	230

SERIES III. *Aluminium.*

$d = 0.603$ *cm.* $b = 74.3$ *cm.*

Temperature Interval.	Bands Counted.	Correction.	Total.	$e \times 10^{-7}$
0 to − 40	17.3			
− 40 " − 80	16.2			
− 80 " −120	12.6			
−120 " −160	10.3			
−160 " −187	2.7			
−187 " −160	2.5	5.4	8.0	134
−160 " −120	10.4	4.1	14.4	163
−120 " − 80	13.8	2.3	15.7	178
− 80 " − 40	16.0	1.5	17.6	199
− 40 " 0	17.8	1.1	18.7	212
0 " 40	18.4	0.8	19.2	217
40 " 99.3	29.7	0.4	30.6	234

In series II. and III. the total number is the sum of the correction and the mean value of the bands counted for that interval as the temperature was falling as it was rising.

The mean of the results of the three series are plotted in Fig. 4. Taking the value from the plot we find e for 40° C. to be about $225 \cdot 10^{-7}$. Fizeau gives $2313 \cdot 10^{-8}$ for 40°, $2336 \cdot 10^{-8}$ for 50° and $315 \cdot 10^{-7}$ for 600°. Tutton has determined e for aluminium, finding it to be $220 \cdot 10^{-7}$, the next figure varying, and an increment per degree rise in temperature of $10 \cdot 10^{-10}$. The values gotten from this and those taken from the curve are quite close for a range of temperature of a few degrees on either side of zero.

Below are given some results for silver.

$d = 1.318$ cm.

$b = 74.4$ cm.

SERIES IV. *Silver.*

Temperature Interval.	Bands Counted.	Correction.	Total.	$e \times 10^{-7}$
0 to − 50	43.0			
− 50 " −100	38.0			
−100 " −150	30.0	9.0	39.0	163
−100 " − 50	41.5	5.0	44.8	187
− 50 " 0	43.7	3.1	46.5	193
0 " 50	44.3	2.2	46.5	193
50 " 99.3	43.4	1.5	44.9	189

SERIES V. *Silver.*

Temperature Interval.	Bands Counted.	Correction.	Total.	$\epsilon \times 10^{-7}$
0 to − 50	43.2			
− 50 " −100	38.8			
−100 " −150	31.8			
−150 " −184	12.8			
−184 " −150	11.8	12.2	24.5	151
−150 " −100	31.2	9.0	40.5	169
−100 " − 50	37.4	5.0	43.1	179
− 50 " 0	43.6	3.1	46.5	193
0 " 50	43.3	2.2	45.5	189
50 " 99.3	43.4	1.5	44.9	189

The results in the two sets of observations, IV. and V., check fairly well, and in both cases the value of ϵ above zero falls somewhat below that between zero and − 50°. This seems to indicate that the point of inflection of the curve, Fig. 3, is considerably

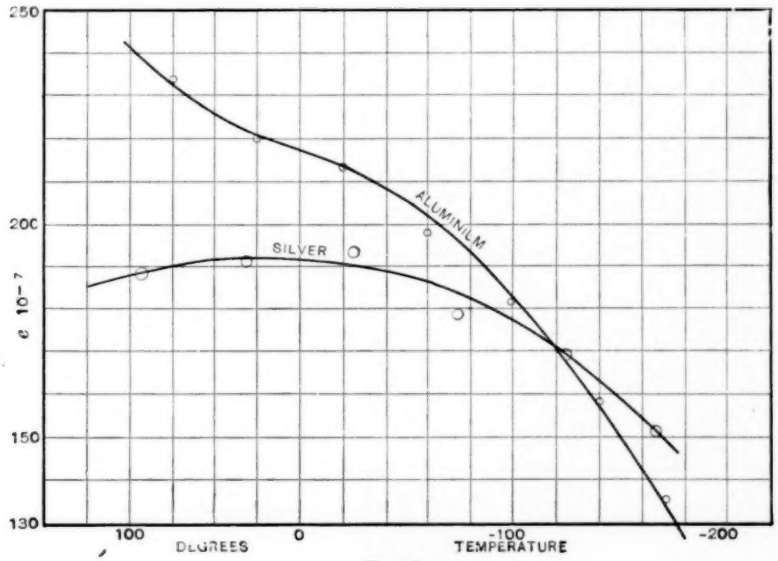

Fig. 3.

above zero. It may be due however to some inaccuracy in the thermo-coil, since a variation of a tenth of an ohm means a shift of almost a band.

The resistance in the case of both coils was not found to be a linear function of the temperature and this makes the determina-

tion of the temperature, especially the lower temperatures, somewhat uncertain.

The lengths of the metal spools have also been measured only roughly, to the third decimal place, by means of a micrometer eyepiece, and the change in length, that is d in formula (6), has been ignored except in the case of the silver spool at temperatures below — 100°.

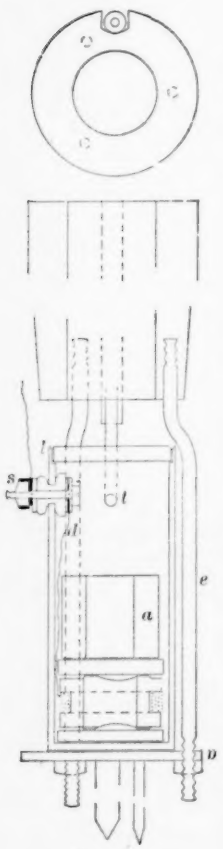

Fig. 4.

It appears from the above data that at low temperatures the refractive index of the air enters as a very important factor, making a correction of more than one hundred per cent. at temperatures somewhat below — 150°. It was desired to check the results obtained by eliminating the correction and to this end work was undertaken *in vacuo*. The only modification made was in the apparatus shown in Fig. 1. A brass cup similar to the one described was made and the top closed by means of a piece of plane plate glass. This was ground to fit in the mouth of the cup, the cup being cut out slightly so as to leave a ledge to prevent the glass being pressed in too far (Fig. 4, *l*). Fish glue was applied to the edge of the glass when it is inserted. Near the top of the cup was soldered a small copper tube, *t*, by means of which connection was made to the air pump. The lead-wires were very short, extending just above the lead weight. Connection was then made to wires outside by means of small bolts passing through rubber stoppers, *s*. The cup was attached to the wooden tube by means of three slender brass bolts, about 15 cm. long. These screwed into the end of the wooden tube and had nuts on their lower ends.

Pulfrich has shown by means of the expression

$$k = 2\frac{d}{\lambda_1}\cdot\frac{\mu_2 - \mu_1}{\mu_1},$$

after making two approximations, that the number of transited bands in the passage from the air-filled space to vacuum is equal to the number of millimeters which the air film is thick. This interesting statement was verified with the apparatus.

The work has not been carried far enough as yet to prove conclusively the correctness of the results obtained, using the correction, but the evidence so far seems to indicate that the correction holds for low temperatures.

I wish to extend my sincere thanks to Professor E. L. Nichols for his interest and suggestions, and to Professor J. S. Shearer, at whose suggestion and under whose direction this work was undertaken and pursued.

PHYSICAL LABORATORY OF CORNELL UNIVERSITY,
June, 1904.

NOTE ON COEFFICIENTS OF EXPANSION AT LOW TEMPERATURE.

By J. S. Shearer.

IN the determination of the expansion coefficients of silver and aluminium at low temperatures by Mr. Ayres it appears that the correction for change of index of the air is large in the lower temperature intervals and is of course greater the greater the length of the specimen used. Mr. Ayres did not have time to fully satisfy himself that the results in vacuum were the same as those found using Pulfrich's formula for the correction. Professor F. M. Simpson, at the writer's suggestion, took up the work during the summer of 1904, with the apparatus used by Ayres, slightly modified for greater convenience.

If $(\mu - 1)/d$ is constant it is evidently immaterial whether we remove the air or simply close the vessel filled with *dry* air at any convenient pressure, so long as we do not liquefy the enclosed gas. It was found convenient to work with air in the enclosure at about 5 mm. pressure and the results demonstrated very clearly that the correction formula was applicable.

The opposite table gives a comparison of the values obtained by these observers for silver and aluminium. The agreement is striking except for the lowest temperature interval in the case of aluminium, where there is quite a discrepancy. Further study in this line is now being prosecuted by Mr. H. G. Dorsey, with apparatus modified in a few details where experience has indicated liability to interruption of work by frosting of surfaces or failure to remain air-tight at extremely low temperatures.

Since the completion of the work described above, the writer has noted a paper by Scheele (Zeitschrift fur Instrumentenkunde) describing a similar method for work in low temperature ranges, in which is given a reference to the inaugural dissertation of Reimerdes, who proposed the ring method in 1896. No reference to this work had previously come to my attention and hence it was not mentioned in Mr. Ayres' paper.

Silver.

Ayres.		Simpson.	
Temp.	E. (Mean).	Temp.	E. (Mean).
0 to − 50	193×10^{-7}	0 to − 40	193×10^{-7}
− 50 " −100	183×10^{-7}	− 40 " − 80	187×10^{-7}
−100 " −150	166×10^{-7}	− 80 " −120	178×10^{-7}
−150 " −184	151×10^{-7}	−120 " −160	165×10^{-7}
		−160 " −180	154×10^{-7}

Aluminium.

Ayres.		Simpson (only one series).	
0 to − 40	216.6×10^{-7}	0 to − 40	216×10^{-7}
− 40 " − 80	198×10^{-7}	− 40 " − 80	199.5×10^{-7}
− 80 " −120	182×10^{-7}	− 80 " −120	190.5×10^{-7}
−120 " −160	158×10^{-7}	−120 " −160	160.5×10^{-7}
−160 " −185	136×10^{-7}	−160 " −180	153×10^{-7}

PHYSICAL LABORATORY, CORNELL UNIVERSITY.

November, 1904.

ELECTRIC DOUBLE REFRACTION IN CARBON DISULPHIDE AT LOW POTENTIALS.

By Gustaf W. Elmén.

WHEN light polarized at an angle of forty-five degrees to the lines of force is passed through certain dielectrics, between two parallel electrodes, they become double-refracting. The difference of phase, δ, between the two components of light at right angles and parallel to the lines of force, as represented by Kerr, and verified by later investigators, is

$$\delta = \pm \frac{BP^2 l}{a^2} \tag{1}$$

where B is the electro-optic constant, depending on the dielectric, l the length and a the distance between the plate electrodes in centimeters, and P the difference of potential between the electrodes in C.G.S. units.

From data obtained in this investigation, there seems to be a decided variation from the above law for low potentials. As the potential was decreased from about 200 volts per millimeter of distance between the electrodes, the decrease of δ was in a smaller ratio than the decrease of P^2.

In determining the electro-optic constant, either a high potential or a sensitive system of measuring δ must be used. In the investigations of Kerr,[1] Quincke,[2] Lemoine,[3] and Schmidt,[4] high potentials were employed. The difference of phase was measured by means of a Babinet's compensator, and the potential, supplied either by a static machine or an induction coil, by means of an electrometer. The difference in the values of B obtained by different observers and also of the values obtained by the same observer, as in the case of CS_2, where the values of Quincke vary by one part in five, Schmidt

[1] J. G. Kerr, Phil. Mag. (4), 50, p. 446, 1875; (5), 8, p. 85 and 229, 1879; (5), 9, p. 157, 1880, and (5), 13, pp. 153 and 248, 1882.

[2] G. Quincke, Wied. Ann., 10, p. 729, 1883.

[3] J. Lemoine, Compt. Rend., 122, p. 835, 1896.

[4] W. Schmidt, Ann. d. Physik., 7, p. 142, 1902.

points out as probably due to the difficulty in measuring high potentials with a sufficient degree of accuracy. To eliminate this error he used two sets of electrodes on the same axis of rotation as the beam of light and set at right angles, so that the effect of one set was compensated by that of the other. This, however, gives only relative values, and some other method must be depended upon for absolute measurements.

A sensitive system of determining the difference of phase has been used in this investigation. In place of the Babinet's compensator, an elliptic half-shade polarizing system [1] was employed. With this system as used in this particular problem values of δ equal to .0001 λ could be observed. In measurements taken the lowest values recorded is .00025 λ. This brought the potential that could be used down to less than 100 volts per millimeter of distance between electrodes of 47 cm. length. For CS_2 with the same length of electrodes the smallest value recorded by Quincke is over 3,000 volts per millimeter of distance. The potential was supplied by a storage battery of about 1,400 volts, and as the smallest potential used was less than 200 volts, a range of seven times this voltage was at the writer's disposal. The potential was measured by means of a voltmeter which had been calibrated so that the possible errors for the lowest potentials were less than one per cent.

The half shade system consisted of two thin mica sections. One, the "sensitive strip" S (Fig. 1) placed next to the polarizing Nicol N, covered half of the field and was placed with its principal axis at an azimuth of forty-five degrees to the plane of vibration of the polarized light. The edge of this strip was parallel to the faces of the electrodes, thus making the vanishing line parallel to the greatest dimension of the field of view. When electrodes of greatest length (127 cm.) were used, this strip was placed between the compensator and the analyzer, as the distance was too long to give good definition. The second section, the "compensator" C (Fig. 1), was placed next to the analyzing Nicol

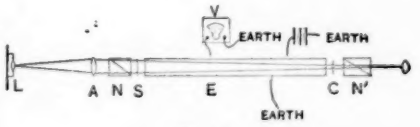

Fig. 1.

[1] D. B. Brace, PHYS. REV., Vol. 18, p. 70, Vol. 19, p. 218, 1904.

N', and covered the whole field. This section was mounted on a circle with a vernier that could be read to minutes. The position of the strip was changed several times so that readings were obtained from different parts of the circle and thus eliminated any possible instrumental errors.

A forty-eight candle power Nernst "glower" one centimeter long and one millimeter in diameter was first used as light-source L (Fig. 1). Later, observations were taken with sunlight passed through a spectroscope and observations were taken for red ($620\,\mu\mu$), green ($540\,\mu\mu$) and blue ($490\,\mu\mu$) light. By means of a condensing lens A (Fig. 1) the light was passed through the polarizing Nicol, the "sensitive strip," the liquid between the plate electrodes E (Fig. 1), through the compensator, analyzer, and thence to a focus O within the observer's eye. When sunlight was used the light was strong enough to allow the use of a short focus telescope with small magnifying power placed next to the analyzer. One of the electrodes was grounded and the other connected to the storage battery. V is a voltmeter, connected across the electrodes.

A thermometer was placed in the liquid and read before and after observations were taken. The measurements were taken at room temperature.

The order of the compensator was found by comparing it with a quarter-wave plate. The compensator was set for a match, and the quarter-wave plate placed in the field so that it produced no effect. The compensator was now rotated through an angle of five or ten degrees and the quarter-wave plate rotated until the intensities of the two halves of the field were again the same. Then, if N', θ' and N, θ are the orders and the degrees respectively through which the two plates are rotated

$$N'\theta = N\theta' \quad (4)$$

[1]

or

$$N = \frac{N'\theta}{\theta'}.$$

Having thus found the order of the "compensator" for a certain wave-length, its order for any other wave-length was found by means of the curve of differential dispersion of mica.[2]

[1] D. B. Brace, PHYS. REV., vol. 18, p. 73, 1904.
[2] E. J. Rendtorff, Phil. Mag., May, 1901, p. 545.

The wave-length for which the quarter-wave plate produced a retardation of $\lambda/4$ was determined in the following manner: A beam of sunlight polarized at forty-five degrees to the principle axis of the plate was passed twice through the plate and then through a second Nicol with its plane parallel to the polarizer. The beam was analyzed by means of a spectroscope. The spectrum contained a black band corresponding to a retardation of half a wave-length. The plate used was found to correspond to $\lambda = 560\ \mu\mu$. In Fig. 2, A and A' are the two Nicols, B the quarter-wave plate, C a mirror silvered on the front surface, and S the spectroscope.

Fig. 2.

In taking observations "the compensator" was set for a match, and then the plate electrodes charged and the "compensator" rotated until a match was again obtained. Then if θ is the angle through which the "compensator" was turned, N the order of the "compensator"

$$\delta = \frac{4N\theta^\circ}{180^\circ} = \frac{N\theta^\circ}{45^\circ} \tag{2}$$

when θ is small and from (1)

$$\frac{N\theta^\circ}{45^\circ} = \pm \frac{BP^2l^2}{a^2}. \tag{3}$$

For the same potential the settings of the mica "compensator" were approximately the same for all the colors used, showing that for the small differences of phase produced the electric differential double refraction of CS_2 is approximately the same as the differential double refraction of mica.

Two lengths of plate electrodes were used. The first was made of nickel-plated brass strips 47 cm. long and 1.5 cm. wide. They were connected to the wires leading to the ground and the battery by two rods, screwed into the plates and taken out through glass tubes 5 cm. long welded on the sides of the containing tube. Small glass strips were placed between them at the edges at intervals and held in position by fish glue. The electrodes were placed in a tube 2.5 cm. in diameter and extended 1 cm. from each end. On the

ends of the tube were fastened squares of glass in which holes 1 cm. in diameter had been bored, and then covered with thin cover-glasses. These cover-glasses were tested for double refraction after they were glued on, for it happened in several cases that the drying of the glue produced enough strain in the glass to be observed.

It was found quite difficult to obtain perfectly parallel metal strips for the longer electrodes, viz., 127 cm. so that plate glass strips were used instead. A thick coating of silver was deposited on one side of each plate, and the silvering carried over at the opposite ends, where contact was made by a couple of brass clamps which also served to hold the plates in position. Copper wires were soldered to these clamps and taken out through holes bored in the end plates of the tube. The electrodes were separated in the same way as before. They rested without support in the tube containing the liquid. Fig. 3, shows a vertical and an end view of the electrodes. The formation of A_2S when the electrodes were in contact with CS_2 was slow, and they could be used for some time without resilvering.

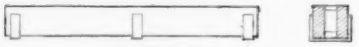

Fig. 3.

The liquid used was CS_2. The ordinary commercial CS_2 was found to have a great number of small particles in it which would vibrate between the electrodes when the charge was put on so that it was found very difficult to maintain a high difference of potential between them. It was therefore found necessary to clarify the liquid by filtering it through a porous cup which was fastened into the neck of a glass flask by means of plaster of Paris. Immediately below the neck was a hole in the flask into which a glass tube was cemented. This tube was connected to a vacuum pump, and thus the liquid was forced through the porous cup. The filtering of the CS_2 did not however seem to appreciably change the electro-optic constant of the liquid. Table I. gives the manner in which each recorded values of B in the following tables were obtained. For voltages below 200 volts per millimeter distance between electrodes each value of B is the mean of twenty observations instead of nine. Tables II. and III. give mean values obtained with the brass electrodes with $a = .2545$ cm. and $a = .184$ cm. Table IV. gives values obtained with the silvered electrodes. The values between

TABLE I.

$l = 47$ *cm.* $a = .2545$ *cm.*

Temp.	Volts.	Read. of Comp. with Pot. On.	Read. of Comp. with Pot. Off.	$B \times 10^{-7}$
19° C.	1360	354° 17′	6° 13′	
		354 38	6 18	
		354 15	6 5	
		354 32	6 16	
		354 16	6	
		354 19	5 54	
		354 15	5 58	
		354 17	5 51	
20° C.		354 24	6 6	
	Mean.	354 22.5	6 4.5	
		Rotation = 11° 42′		4.05

TABLE II.

$l = 47$ *cm.* $\lambda = 560\ \mu\mu$. $a = .2545$ *cm.*

Temp.	Volts.	o°	$B \times 10^{-7}$	Temp.	Volts.	o°	$B \times 10^{-7}$
23.5° C.	1389	11.66°	3.88	20°	357	1.1°	5.53
19.5	1360	11.68	4.05	19	324	.74	4.52
18.5	1345	11.6	4.11	20	324	.84	5.13
20	1043	6.77	3.94	19	283	.76	6.04
19	995	6.28	4.07	24.5	252	.65	6.77
20	880	4.74	3.94	22	252	.68	6.56
21	720	3.16	3.92	22	247	.62	6.51
22	649	2.58	3.93	19.5	244	.77	8.29
19	556	2.16	4.53	19	243	.53	5.88
22	531	1.8	4.09				

TABLE III.

$l = 47$ *cm.* $\lambda = 560\ \mu\mu$. $a = .184$ *cm.*

Temp.	Volts.	o°	$B \times 10^{-7}$	Temp.	Volts.	o°	$B \times 10^{-7}$
23° C.	876	8.7°	3.78	23°	193	0.58°	5.19
25	861	8.24	3.71	25	191	0.53	4.56
23	418	2	3.82	25	191	0.46	4.23
25	242	.74	4.22				

TABLE IV.

$l = 127$ *cm.* $\lambda = .560\ \mu\mu$. $a = 656$ *cm.*

19°	1364	4.84°	4.08	18°	1314	4.53°	4.11
20	1355	4.88	4.17	20	1017	2.8	4.25
20	1334	4.38	3.85	20	1017	2.87	4.35
20.5	1328	4.27	3.84	19.5	1040	2.9	4.17

the three sets agree fairly well. Table V. gives values obtained for

TABLE V.

$l = 47$ *cm.* $a = .184$ *cm.*

λ in μμ.	Temperature.	Volts.	θ°	$B \times 10^{-7}$
620	23°	807	7.6	3.00
	23	525	3.1	2.93
	23	240	.73	3.17
	23	190	.6	4.32
540	23	997	11.75	4.13
	23	841	8.5	4.16
	23	543	3.4	4.20
	23	242	.78	4.88
	23	191	.6	5.96
490	23	955	11.5	5.51
	23	670	5.38	5.64
	26	387	1.67	5.23
	26	250	.66	5.33
	26	190	.5	6.39

different colors. For the same value of P for different wave-lengths, θ remains approximately the same, and in calculating the value of B

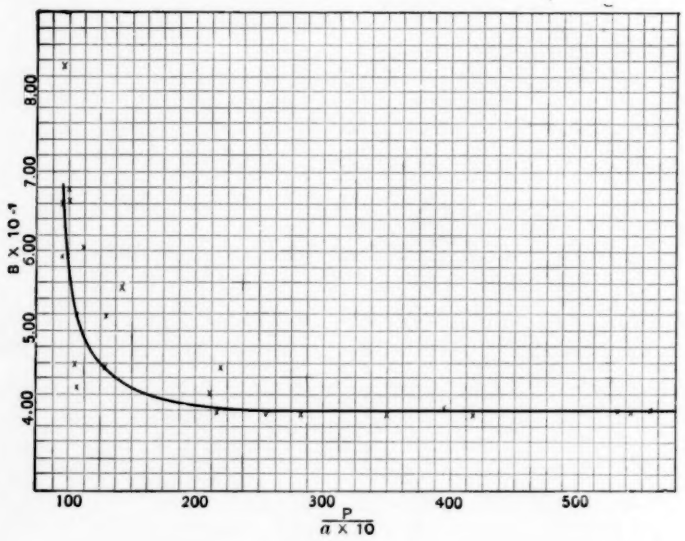

Fig. 4.

it was only necessary to multiply the value of B, obtained from equation (3), by the ratio of the order of the "compensator" ($N =$

1/43.8) determined by means of the quarter wave-plate for that particular value of λ, which, in this case, was 560 $\mu\mu$, while the order for any other colors used was obtained by interpolating from the curve obtained by Rendtorff.

In Fig. 4 the values of $B \times 10^{-7}$ are plotted as ordinates and $P/a \times 10$ as abscissæ. The curve is approximately a straight line until about 200 volts per millimeter of distance, when it turns and seems to become almost asymptotic to the ordinates. On obtaining these low values the errors of observation were of course, large, owing to the smallness of the effect, and at first it was thought that the large deviation from the law was due to some systematic error. To eliminate such possible errors, the apparatus was taken down and remounted under different conditions. This however did not give any difference in values obtained for B.[1]

There is a great similarity between B, the constant of electric polarization in CS_2, as determined by means of double-refraction, and μ the permeability, as determined by means of magnetic induction, and this suggests that possibly the double-refraction in liquids under electric strain is due to a state of polarization analogous in nature to that which takes place in a magnetic substance when in a state of polarization.

In conclusion the writer desires to thank Professor Brace for his valuable suggestions which made this investigation possible.

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[1] It might be suggested that the readings of the voltmeter do not give the true drop in potential within the liquid owing to a possible polarization at the electrodes, analogous to that within electrolytes, which, at the low potentials used, might make itself evident. This could be determined by varying the distance of the electrodes and comparing the observations for the same drop of potential. This conclusion does not seem to be borne out by the data in tables II. and III. where $a = .2545$ cm. and .184 cm. respectively.

the three sets agree fairly well. Table V. gives values obtained for

TABLE V.

$l = 47$ *cm.* $a = .184$ *cm.*

λ in μμ.	Temperature.	Volts.	o°	$B \times 10^{-7}$
620	23°	807	7.6	3.00
	23	525	3.1	2.93
	23	240	.73	3.17
	23	190	.6	4.32
540	23	997	11.75	4.13
	23	841	8.5	4.16
	23	543	3.4	4.20
	23	242	.78	4.88
	23	191	.6	5.96
490	23	955	11.5	5.51
	23	670	5.38	5.64
	26	387	1.67	5.23
	26	250	.66	5.33
	26	190	.5	6.39

different colors. For the same value of P for different wave-lengths, θ remains approximately the same, and in calculating the value of B

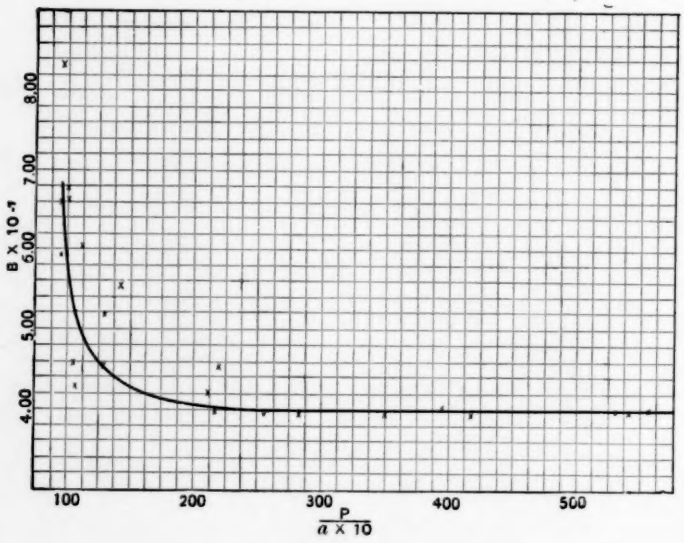

Fig. 4.

it was only necessary to multiply the value of B, obtained from equation (3), by the ratio of the order of the "compensator" ($N =$

1/43.8) determined by means of the quarter wave-plate for that particular value of λ, which, in this case, was 560 $\mu\mu$, while the order for any other colors used was obtained by interpolating from the curve obtained by Rendtorff.

In Fig. 4 the values of $B \times 10^{-7}$ are plotted as ordinates and $P/a \times 10$ as abscissæ. The curve is approximately a straight line until about 200 volts per millimeter of distance, when it turns and seems to become almost asymptotic to the ordinates. On obtaining these low values the errors of observation were of course, large, owing to the smallness of the effect, and at first it was thought that the large deviation from the law was due to some systematic error. To eliminate such possible errors, the apparatus was taken down and remounted under different conditions. This however did not give any difference in values obtained for B.[1]

There is a great similarity between B, the constant of electric polarization in CS_2, as determined by means of double-refraction, and μ the permeability, as determined by means of magnetic induction, and this suggests that possibly the double-refraction in liquids under electric strain is due to a state of polarization analogous in nature to that which takes place in a magnetic substance when in a state of polarization.

In conclusion the writer desires to thank Professor Brace for his valuable suggestions which made this investigation possible.

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[1] It might be suggested that the readings of the voltmeter do not give the true drop in potential within the liquid owing to a possible polarization at the electrodes, analogous to that within electrolytes, which, at the low potentials used, might make itself evident. This could be determined by varying the distance of the electrodes and comparing the observations for the same drop of potential. This conclusion does not seem to be borne out by the data in tables II. and III. where $a = .2545$ cm. and .184 cm. respectively.

NEW BOOKS.

Laboratory Physics. By D. C. MILLER. Boston, Ginn & Company, 1903. Pp. xv + 404.

This laboratory manual is designed to be a hand-book for the performance of quantitative experiments in general physics for students in colleges and technical schools. It is presumed that the student has already had an elementary course in laboratory physics. The work for which this manual is written is supposed to be accompanied by lectures and recitations in general physics.

The work is based on the author's experience of twelve years, and many of the experiments have been used successfully for the past six years with practically no change in their forms. In all one hundred twenty-eight exercises are described.

In the introduction, in addition to the usual directions regarding details of the work, the author treats briefly of observational errors and corrections, probable error, least squares, weighing observations, and the graphical interpretation of results. Unless the student has already had some work on curve plotting and interpretation it might seem advisable to extend considerably the treatment of this part of the notes since this method of representing data and results is becoming more general and is so wide in its application.

Although the manual is based on the work done in a particular laboratory it is unlike most manuals based on similar experience in that the exercises are written up in such general terms as to make them readily applicable to any laboratory with varying details in the apparatus at hand. It may be, however, that additional notes and directions will be needed in some cases.

Many bits of general information useful to the student and instructor alike, are given. Another valuable feature of the book is that references are freely given, to original sources or to those sources in which the desired information may be found in the most desirable form.

In an appendix are gathered numerous useful physical constants, a list of general reference books, and tables. The tables of logarithms and natural trigonometrical functions would seem to be too much abbridged to be most useful.

The book work is good and the exercises are illustrated by many drawings and diagrams. The manual will be found useful as a work of reference as well as a laboratory guide. ERNEST BLAKER.

A Text-Book of General Physics. By JOSEPH S. AMES. American Book Company, 1904. Pp. i + 668.

This volume is in no sense a revision of Professor Ames' well-known "Theory of Physics." It is essentially a new text-book and does not duplicate the earlier work in form of statement or illustration. The newer work although larger is not of a more advanced grade than the original. The treatment is, if anything, more elementary. Only the simplest relations are given mathematical expression and in general the grade of difficulty is not above that which the common experience of teachers finds adapted to the lower classes in the college.

The combination of simplicity with accuracy of statement is the essential feature of a practicable book for use with beginners in college, and it may justly be said of Professor Ames' volume that it posesses this combination of qualities to an unusual degree. The "Physics" is that of a working physicist reduced to its simplest terms, and the selection of material is such as to afford excellent preparation for more advanced work.

E. L. N.

Frick's Physikalische Technik: siebente Auflage. Von Dr. OTTO LEHMANN. Braunschweig, Viewig & Sohn, 1904. Pp. xiii + 630.

When Dr. Frick's "Physikalische Technik" first appeared in 1856 it met with a hearty reception on the part of his fellow teachers of science and of everyone interested in experimental physics. Several editions were called for to meet the demand and the book was doubtless an important factor in bringing to an end the unfortunate period during which the only recognized apparatus for the teaching of science in the schools was the blackboard and chalk.

After the author's death the work of revision was undertaken by his successor, Professor Reichet, who brought out the fifth edition. In 1883, Professor Otto Lehmann, of the Technical High School in Karlsruhe, became the editor.

With the development of science teaching and the corresponding growth of the art of the instrument maker the nature and contents of such a handbook were necessarily changed and its scope greatly enlarged. The seventh edition, while it bears the original title, is practically a new work consisting of two large volumes. The first part of volume I., which contains 630 pages with over two thousand illustrations, deals with the construction and equipment of the physical laboratory.

In the first chapter the author considers briefly the nature of the building itself. The second chapter, which is the longest, is devoted to the arrangement and equipment of the lecture room, beginning with its architecture and including every detail involved in the planning of a modern and complete equipment for demonstration. How complicated

an affair the modern lecture room for science teaching has become may be gathered from the fact that this chapter has fifty-four distinct headings.

In the remaining three chapters of part I. the arrangement of preparation rooms, apparatus rooms, workshops, offices, etc., are discussed in equally admirable detail. Nothing seems to have been forgotten, even to the devices for filing away and preserving pamphlets, catalogues and written matter. While the materials gathered in this work are directly applicable to the conditions existing in Germany, it will be of invaluable assistance to everyone who is confronted with the task of planning and equipping a physical laboratory. E. L. N.

Lehrbuch der Physik. O. D. CHWOLSON. Ubersetzt für H. PFLAUM, Vieweg, Braunschweig, 1904. Pp. xxii + 1056.

The second volume of Chwolson's treatise on physics deals with the subjects of sound and radiation. The author's treatment of his subjects is characterized throughout by simplicity and clearness and there is as near an approach to completeness as the enormous mass of material which has accumulated in later years will permit within the limits of a work of this kind. To nearly every important research reference is made although of necessity in many cases the author has been compelled to confine himself to a mere mention of the results obtained and to omit all details. Under each important topic, however, the reader will find a historical sketch, the reading of which will greatly facilitate his subsequent search through the original sources and at the end of each section the list of authorities is sufficiently complete to serve its purpose. In these days when the literature of every portion of the science has grown to appalling volume a work of this description is invaluable.